Iterative Linear State Estimation Using a Limited Number of PMU Measurements

Ramtin Khalili, Ali Abur
Department of Electrical Engineering
Northeastern University
Boston, MA, USA
{rkhalili, abur}@ece.neu.edu

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***Abstract*—State estimation (SE) in transmission systems is commonly carried out based on the supervisory control and data acquisition (SCADA) system measurements. The solution involves a recursive process due to the non-linearity of the measurement equations. However, the SCADA-based SE is prone to ill-conditioning which may lead to convergence issues. This paper presents an alternative and computationally robust method which uses primarily SCADA measurements plus a limited number of voltage measurements provided by phasor measurement units (PMUs). This is accomplished by first converting the power flow measurements into equivalent current phasors followed by a linear state estimation solution. Instead of updating the jacobian and gain matrix in each iteration, in this approach, the current phasors will be updated using the estimated phase angles from the previous iteration. Numerical examples will be presented to illustrate the performance of the method when applied to typical power systems.**

***Index Terms*—Linear State Estimation, Phasor Measurement Units, SCADA measurements**

I. Introduction

State estimation (SE) has become one of the critical applications in today's energy management systems. While phasor measurement units (PMUs) are rapidly populating power grids, most state estimators still rely primarily on SCADA measurements which provide not only full observability but also sufficient redundancy to enable bad data detection and identification. SCADA measurements are received every few seconds and SE is executed every few minutes during normal operation. Given the non-linear nature of the SCADA measurement equations, the SE solution is obtained via iterative gradient-based methods as documented in different publications [1]–[4].

Solution of non-linear algebraic equations commonly suffer from poor initialization and/or numerical ill-conditioning impacting the convergence rate and solution accuracy. In the case of state estimation problem, introduction of PMUs enabled simplification of the problem formulation and solution due to the linearity of PMU measurements with respect to system states [5], [6]. Hence, the SE solution can be found directly (non-iterative) if there are sufficient PMU measurements to render the entire power grid observable [7]–[10].

Unfortunately, most power systems still lack sufficient number of PMUs to make the system observable by only PMU measurements. As an alternative, hybrid state estimation algorithms that can utilize both SCADA and PMU measurements have been investigated by several groups. The proposed solutions and discussion of different implementation challenges can be found in the literature [11]–[18]. Generally, these methods can be classified under two categories based on the way they incorporate PMU measurements into the existing SCADA measurements: 1) hierarchical and 2) simultaneous.

In the hierarchical methods [11]–[14], the main idea is to execute the SCADA-based SE first and then improve the final estimated states by incorporating the PMU measurements next as a second stage. On the other hand, the methods in the second category [15]–[18] process both types of measurements simultaneously in a central estimator. Recently, a number of linear SCADA-based SE methods have been proposed in several papers [19]–[23]. They report drastic reductions in computation times but require significant reformulation of the existing SE algorithms and some of them have implicit limitations.

In this paper, an iterative linear state estimation (LSE) is proposed by transforming conventional power measurements into equivalent current phasors. The transformation requires information on bus voltage phase angles, which are not known unless a PMU is installed at that bus. It is shown that having PMUs installed at less than 4% of the system buses is sufficient to implement the proposed approach for a given system. This method is a hybrid LSE method that is easy to implement yet provides significant improvements in the CPU time in comparison with the conventional methods. Although the proposed method is still iterative, each iteration carries a very low computational cost due to the constant coefficient matrices used repeatedly during the iterations. Note that PMU and SCADA measurements are both treated in the same way after the measurement conversion. Once the states are estimated, the measurement transformation will be done again with the estimated phase angles. Then, the states will be re-estimated using the updated measurements and this task will be repeated until convergence tolerance is reached.

It is noted that the idea of transforming power expressions

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into equivalent currents is not new and initially proposed for power flow problem [24], [25] and then also applied to state estimation [26]. However, when applied to the problem of state estimation, it is observed that robustness of the approach strongly depends on proper initialization which is addressed via the use of limited number of PMU measurements in this paper.

II. Conventional State Estimation Review

State estimation is the problem of obtaining the voltage magnitudes and phase angles at all buses using the measurements and exact network model. For this purpose, considering SCADA measurements the nonlinear measurement model is expressed as follows [1]:

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{X}) + \boldsymbol{e} \tag{1}$$

where:

$\boldsymbol{z}$	$m \times 1$ measurement vector
$\boldsymbol{h}(.)$	$m \times 1$ vector of measurement functions
$\boldsymbol{X}$	$n \times 1$ state vector $[\boldsymbol{\theta}^T, \lvert\boldsymbol{V}\rvert^T]^T$
$\boldsymbol{e}$	$m \times 1$ measurement error vector
n	Number of states
m	Number of measurements

In the sequel, boldface variables will be used to denote matrices or vectors. The problem of estimating the state vector $\boldsymbol{X}$ can be transformed into an optimization problem where some norm of the difference between the recorded and estimated measurements is minimized. So, the cost function J which is the weighted sum of residuals, is defined as follows:

$$J(\hat{\boldsymbol{X}}) = [\boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{X}})]^T \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{X}})] \tag{2}$$

where $\hat{\boldsymbol{X}}$ is the estimated states and $\boldsymbol{R}$ is the covariance of measurement errors defined as:

$$\boldsymbol{R} = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_m^2) \tag{3}$$

In order to minimize the cost function, the first-order optimality condition $\frac{\partial J(\boldsymbol{X})}{\partial \boldsymbol{X}} = 0$ is applied to (2). This yields the following iterative solution through the Gauss-Newton method:

$$\hat{\boldsymbol{X}}^{i+1} = \hat{\boldsymbol{X}}^i + [\boldsymbol{G}(\hat{\boldsymbol{X}}^i)]^{-1} \boldsymbol{H}^T(\hat{\boldsymbol{X}}^i) \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{X}}^i)] \tag{4}$$

where i is the iteration number, $\boldsymbol{H}$ represents the $m \times n$ measurement jacobian, and $\boldsymbol{G}$ denotes the gain matrix which is defined as follows:

$$\boldsymbol{G}(\hat{\boldsymbol{X}}^i) = [\boldsymbol{H}^T(\hat{\boldsymbol{X}}^i)] \boldsymbol{R}^{-1} [\boldsymbol{H}(\hat{\boldsymbol{X}}^i)] \tag{5}$$

So, $\hat{\boldsymbol{X}}$ will be obtained using (4) and (5) recursively until the difference between two consecutive iterations becomes lower than a predetermined threshold. As can be seen in (4) and (5), estimated measurements ($\boldsymbol{h}(\hat{\boldsymbol{X}})$), jacobian, and the gain matrix should be calculated and updated in each iteration which is computationally expensive. For more details on the conventional SE formulation based on SCADA measurements see [1]–[4].

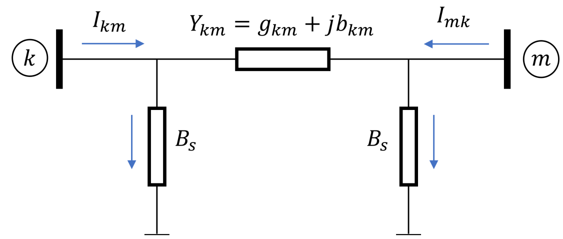

Fig. 1. Pi-equivalent of a transmission line.

III. Linear State Estimation

Typically the measurement vector $\boldsymbol{z}$ in (1) contains SCADA-type measurements including active and reactive powers and voltage magnitudes. Hence, $\boldsymbol{h}$ is a nonlinear function and the state estimation problem requires an iterative solution as discussed in the previous section. If voltage and equivalent current phasors can replace the power flows and injections (as in the case of PMU measurements), (1) can be formulated as a linear equation with a direct (non-iterative) solution. In this section, first, linear state estimation is formulated based on the assumption that voltage and current phasors are available. Given the fact that the number of installed PMUs are limited, a new framework is proposed to transform the SCADA measurements into PMU-type measurements in subsection III-B in order to use the LSE formulation. Challenges associated with the initialization of this transformation are also discussed in subsection III-C.

A. State Estimation based on Current Phasor Measurements

Based on the π-equivalent of the transmission lines which is shown in Fig. 1, current flowing through bus k to m can be written in terms of the line admittance and voltages at both ends as follows:

$$I_{km} = (V_k - V_m)Y_{km} + jB_sV_k \tag{6}$$

where, Y_{km} and B_s are the series admittance and line charging susceptance of the line $k-m$. By substituting $Y_{km} = g_{km} + jb_{km}$, real and imaginary parts of I_{km} can be detached as follows:

$$Re\{I_{km}\} = g_{km}(Re\{V_k\} - Re\{V_m\}) - b_{km}(Im\{V_k\} - Im\{V_m\}) - B_s Im\{V_k\} \tag{7}$$

$$Im\{I_{km}\} = g_{km}(Im\{V_k\} - Im\{V_m\}) + b_{km}(Re\{V_k\} - Re\{V_m\}) + B_s Re\{V_k\} \tag{8}$$

Considering voltage phasors and current phasors as the measurements, voltages and currents can be expressed in the rectangular coordinates as $V = E + jF$ and $I = C + jD$. So, the measurement vector can also be written in the rectangular form:

$$\boldsymbol{z} = \begin{bmatrix} Re\{\boldsymbol{V}\} \\ Im\{\boldsymbol{V}\} \\ Re\{\boldsymbol{I_{km}}\} \\ Im\{\boldsymbol{I_{km}}\} \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}^{meas} \\ \boldsymbol{F}^{meas} \\ \boldsymbol{C}^{meas} \\ \boldsymbol{D}^{meas} \end{bmatrix} \tag{9}$$

where, superscript $meas$ indicates a measurement. similarly, states can be expressed in the rectangular coordinates as the real and imaginary parts (E, F) rather than voltage magnitude and phase angle ($|V|$, θ):

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{F} \end{bmatrix} \tag{10}$$

As can be seen from (7)-(10), the relationship between the measurements and states is now linear which is given by:

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{e} \tag{11}$$

where:

$\boldsymbol{z}$	$m \times 1$ measurement vector defined in (9)
$\boldsymbol{H}$	$m \times n$ Jacobian
$\boldsymbol{X}$	$n \times 1$ state vector $[\boldsymbol{E}^T, \boldsymbol{F}^T]^T$
$\boldsymbol{e}$	$m \times 1$ measurement error vector

Given the linear measurement model in (11), states can be estimated using WLS method as follows:

$$\hat{\boldsymbol{X}} = \boldsymbol{G}^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{z} \tag{12}$$

where, $\boldsymbol{G}$ is the gain matrix defined as:

$$\boldsymbol{G} = \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H} \tag{13}$$

Note that unlike (4) and (5), the solution based on (12) and (13) is not iterative. From the mathematical perspective, the LSE in (12) and (13) is nothing but a linear mapping from the measurements ($\boldsymbol{z}$) to the states ($\hat{\boldsymbol{X}}$). Thus, the LSE can be expressed as the following linear transformation matrix:

$$\boldsymbol{\Gamma} = \boldsymbol{G}^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1} \tag{14}$$

Note that $\boldsymbol{\Gamma}$ remains constant unless there is a change in the network parameters or topology.

B. Transforming the Nonlinear SE Problem to an Iterative Linear Problem

Derivation of (11) is based on the assumption that current phasor measurements are available. While this may be the case for networks which are fully observable by PMUs, most power grids have only a limited number of PMUs short of rendering the system observable all by themselves. Hence, commonly available and redundant SCADA measurements for which the SE problem is nonlinear and the solution is iterative as shown in (4) and (5), need to be incorporated. In order to accomplish this and still make use of the LSE formulation of (12) and (13), all power flow measurements should be converted into equivalent current phasors. Complex power flows measured at the terminals of a given line k-m can be expressed in terms of the terminal bus voltage and current phasors as follows:

$$\begin{aligned} S_{km} &= V_k I_{km}^* \\ S_{mk} &= V_m I_{mk}^* \end{aligned} \tag{15}$$

Letting $S_{km} = P_{km} + jQ_{km}$ and $S_{mk} = P_{mk} + jQ_{mk}$, (15) can be solved for the currents as follows:

$$\begin{aligned} I_{km} &= \left(\frac{P_{km} + jQ_{km}}{V_k}\right)^* \\ I_{mk} &= \left(\frac{P_{mk} + jQ_{mk}}{V_m}\right)^* \end{aligned} \tag{16}$$

where, P_{km}, P_{mk}, Q_{km}, and Q_{mk} are active and reactive power flow measurements which are provided by SCADA system. The missing information in the expressions for the current phasors I_{km} and I_{mk} is related to the phase angles of the voltage phasors $V_k = |V_k|\angle\theta_k$ and $V_m = |V_m|\angle\theta_m$.

This necessitates an iterative solution where an initial guess on the phase angles is made and equivalent current phasor measurements are calculated using (16). A reference is chosen for the assumed phase angles of all buses except for those where PMU measurements and therefore phase angle measurements are readily available. The procedure for selecting the reference is investigated in the next subsection. Once the equivalent current phasors for all power flow measurements (for non-PMU buses) are obtained, the states are estimated using the linear transformation $\boldsymbol{\Gamma}$ derived in (14). Since the reference was chosen arbitrarily, current measurements should be updated using the most recently estimated voltages. The solution algorithm will alternate between linear state estimation and current measurement corrections until the absolute changes in the estimated states drop below a convergence tolerance. Since $\boldsymbol{\Gamma}$ is only calculated once and remains constant during iterations, the CPU time for the overall estimation process is drastically reduced as will be demonstrated in the simulations section below.

C. Choosing the Reference for Measurement Transformation

Choosing a suitable reference angle for initializing the transformation in (16) is critical in the numerical robustness and computational performance of the algorithm. For power grids where the bus voltage phase angles remain close together, this may not have a significant impact on the performance of the algorithm. However, for most large power grids where this is not the case, it will be demonstrated via simulations that the number of LSE iterations will drastically increase by choosing only one reference for the whole system without any phase angle measurement.

One strategy that effectively addresses the initialization problem is proposed and implemented in this work. It assumes that there are a few PMUs installed in the system where the corresponding bus voltage phase angles are measured. Experimental evidence suggests that a small number of PMUs (less than 5% of total number of buses) will be sufficient to successfully implement this approach. Once the buses with PMU installations are determined, for all system buses the minimum distance to the nearest PMU bus is calculated as follows:

$$\begin{aligned} \rho_i &= \min\{\boldsymbol{D}\langle bus_i, bus_j^{pmu}\rangle\} \\ i &= \{1, ..., n_b\}, j = \{1, ..., n_p\} \end{aligned} \tag{17}$$

where:

Bus	$n_b \times 1$ vector of system buses (i^{th} entry: bus_i)
Bus^{pmu}	$n_p \times 1$ vector of PMU buses (j^{th} entry: bus_j^{pmu})
n_b	number of system buses
n_p	number of installed PMUs
ρ_i	min. distance to the nearest PMU bus from bus i

and $\boldsymbol{D}\langle bus_i, bus_j^{pmu}\rangle$ is the minimum number of buses between bus i (including bus i) and j^{th} PMU. So, $\boldsymbol{D}\langle bus_i, Bus^{pmu}\rangle$ is a $n_p \times 1$ vector corresponding to the distance of bus i to all PMUs. Let $\boldsymbol{\phi}$ be the $n_b \times 1$ vector of nearest PMU bus to each bus. Thus, elements of $\boldsymbol{\phi}$ denoted by ϕ_i can be expressed as follows:

$$\phi_i = \{bus_j^{pmu} : j \in \{1, ..., n_p\}, \boldsymbol{D}\langle bus_i, bus_j^{pmu}\rangle = \rho_i\} \tag{18}$$

So, the vector of reference phase angle $\boldsymbol{\Theta}_{ref}$ can be obtained as follows:

$$\boldsymbol{\Theta}_{ref} = \boldsymbol{\Theta}(\boldsymbol{\phi}) \tag{19}$$

$\boldsymbol{\Theta}$ is the phase angles vector where the ϕ_i element is measured by the installed PMUs. To illustrate the procedure, consider the 5-bus system shown in Fig. 2 where two PMUs are installed at buses 1 and 2.

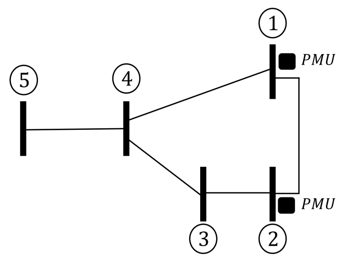

Fig. 2. 5-bus test system.

The vector $\boldsymbol{\phi}$ and the distances to the nearest PMU-bus (ρ_i) for each bus i will be given by:

$$\boldsymbol{\phi} = \begin{bmatrix} 1 & 2 & 2 & 1 & 1 \end{bmatrix}^T$$

$$\rho_1 = 0, \rho_2 = 0, \rho_3 = 1, \rho_4 = 1, \rho_5 = 2$$

The vector of reference phase angles will thus be obtained as:

$$\Theta_{ref} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_2 & \theta_1 & \theta_1 \end{bmatrix}^T$$

Note that the cold start initialization is considered here, where the bus phase angles are unknown except for the buses with installed PMUs. Alternatively, during pseudo-steady-state operation one can take advantage of a warm start by initializing the algorithm using the estimated states from the previous SE run, further improving the accuracy.

D. Summary of the Proposed LSE

Steps of the proposed LSE method can be listed as follows:

Step 1 Form bus admittance matrix and measurement Jacobian using parameters and topology of the network.

Step 2 Specify the appropriate values for the standard deviation of SCADA and PMU measurements.

Step 3 Calculate gain and linear transformation matrices using (13) and (14).

Step 4 Obtain reference phase angle vector ($\boldsymbol{\Theta}_{ref}$) for initialization using (17)-(19).

Step 5 Convert power measurements to currents using (16) and the reference obtained in the previous step. Except the first iteration, measurement conversion will be done using the estimated states of the previous iteration.

Step 6 Form measurement vector as (9).

Step 7 Estimate the states by $\hat{\boldsymbol{X}} = \boldsymbol{\Gamma}\boldsymbol{z}$.

Step 8 If $\left|\hat{\boldsymbol{X}}^k - \hat{\boldsymbol{X}}^{k-1}\right| < \epsilon$ stop, otherwise go back to the step 5.

IV. Simulation Results and Discussion

Several IEEE test systems are considered in this section to test the effectiveness of the proposed LSE method which is summarized in subsection III-D. MATLAB 2019b is utilized for simulations and MATPOWER 7.0 [27] is also employed for generating the power flow solutions for various test cases.

First, IEEE 14-bus system is considered without any installed PMUs. Applying the LSE method yields accurate states for the system through 4 ultra fast iterations. However, without using PMUs and starting with only the slack bus phase angle, the number of iterations increases to 14 and 90 for IEEE 118 and 300-bus systems, respectively. The reason for this phenomenon is shown in Fig. 3 where bus phase angles are represented for IEEE 14, 118, and 300-bus systems. As can be seen in Fig. 3, the phase angles for all buses of IEEE 14-bus system are within a small bound. So, initializing the current transformation with only one reference bus returns acceptable SE results. Yet, bus phase angles for other systems distributed in larger bounds where the difference between the largest and the lowest phase angle reaches roughly 80 degrees for IEEE 300-bus system.

Seemingly, in these cases more phase angle references will be needed. So, a few PMUs are placed at different buses in order to provide adequate known phase angles for current transformation. Table I shows the number of PMUs and the PMU locations for each test system. As shown in Table I, only a few PMUs are needed in this method which is less than 5% of the total number of system buses. Moreover, the number and location of PMUs given in Table I can be further improved since no attempt was made to optimize that choice in this study. However, on-going work is investigating optimal PMU placement for this purpose and the reults will be reported in a future publication. Note that the optimal PMU placement is not only a function of the network size but also the topology as well as the distribution of the voltage phase angles.

Number of iterations and the CPU time of the proposed LSE method are shown in Table II for all the test cases. The CPU time which is reported in Table II is averaged over 100 run of the LSE. All the simulations in this paper are performed on an i7-9750H CPU @ 2.60GHz and 16 GB of RAM laptop. Computation time of this method is much shorter than most of the published methods in the literature for the same systems (for example [23]) due to the following reasons:

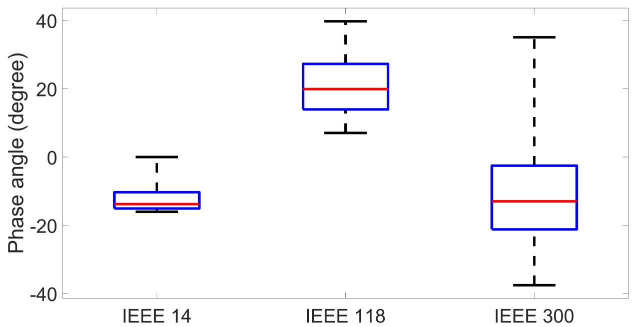

Fig. 3. Test systems phase angles

TABLE I
NUMBER OF INSTALLED PMUS AND THEIR LOCATION

System	No. of PMUs	No. of PMUs /Total buses	PMU buses
IEEE 14	0	0	-
IEEE 118	4	3.4%	[10, 69, 89, 115]
IEEE 300	12	4%	[1, 43, 91, 114, 143, 169, 193, 227, 247, 257, 283, 294]

- The nonlinear SE problem is transformed into the LSE by a computational inexpensive transformation of measurements.
- The jacobian is formed only once and kept constant throughout the iterations.
- Building, updating and factorizing the gain matrix at each iteration (in the conventional methods) is no longer necessary. Instead, a constant gain matrix is formed and factorized only once.
- Individual iterations can be carried out very fast using sparse linear solvers.
- SCADA and PMU-type measurements are effectively reconciled.

In order to assess performance of the LSE method, the absolute error between the estimated and true states is calculated for each state. The average absolute errors ($\boldsymbol{\sigma}$) are also computed for each test system. This index can be written for voltage magnitudes and phase angles separately as follows:

$$\boldsymbol{\sigma}_V = \frac{1}{n_b}\sum_{i=1}^{n_b}\left|\left|\hat{V}_i\right| - \left|V_i^{true}\right|\right| \tag{20}$$

and,

$$\boldsymbol{\sigma}_\theta = \frac{1}{n_b}\sum_{i=1}^{n_b}\left|\hat{\theta}_i - \theta_i^{true}\right| \tag{21}$$

The above indices are calculated for all three systems and reported in Table III. The absolute estimated voltage magnitude and phase angle error for all buses of IEEE 118-bus and 300-bus systems are shown in Figs. 4 - 7.

TABLE II
STATE ESTIMATION ITERATIONS AND CPU TIME

System	No. of iterations	CPU time [s]
IEEE 14-bus	4	0.0014
IEEE 118-bus	4	0.0164
IEEE 300-bus	5	0.0638

TABLE III
STATE ESTIMATION PERFORMANCE INDEX

System	$\boldsymbol{\sigma}_V(pu)$	$\boldsymbol{\sigma}_\theta(deg.)$
IEEE 14-bus	3.56×10^{-7}	2.66×10^{-6}
IEEE 118-bus	1.16×10^{-6}	1.76×10^{-5}
IEEE 300-bus	3×10^{-6}	7.28×10^{-5}

V. CONCLUSION

This paper presents an iterative LSE method considering SCADA measurements and a limited number of PMUs. While the SCADA-based SE is a nonlinear estimation problem requiring an iterative solution algorithm, it is reformulated as an iterative LSE problem. This is accomplished by converting all power measurements into equivalent current phasors at each iteration using the estimated states from the previous iteration. It is shown that the algorithm is highly sensitive to initialization and a simple yet effective initialization strategy is developed and implemented taking advantage of a few PMU measurements. Since the jacobian remains constant during the iterations, the CPU time is improved drastically in comparison with the conventional SE methods. The performance of the proposed method is tested and validated using three different IEEE test systems.

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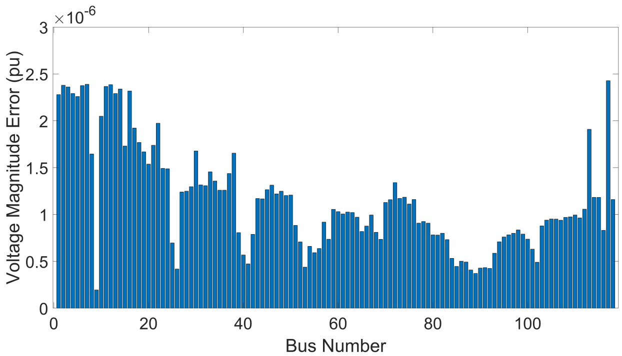

Fig. 4. Voltage magnitudes absolute error in pu for IEEE 118-bus system.

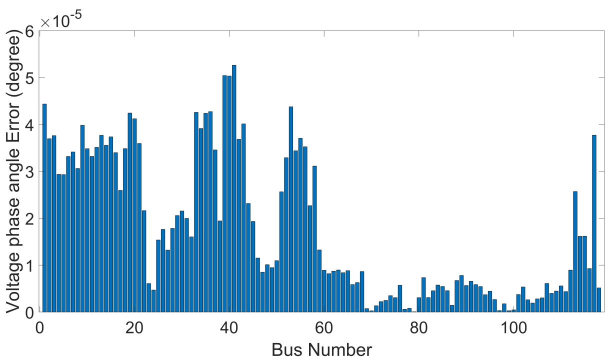

Fig. 5. Phase angles absolute error in degree for IEEE 118-bus system.

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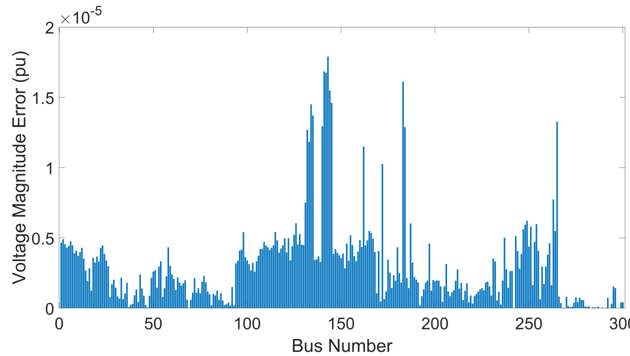

Fig. 6. Voltage magnitudes absolute error in pu for IEEE 300-bus system.

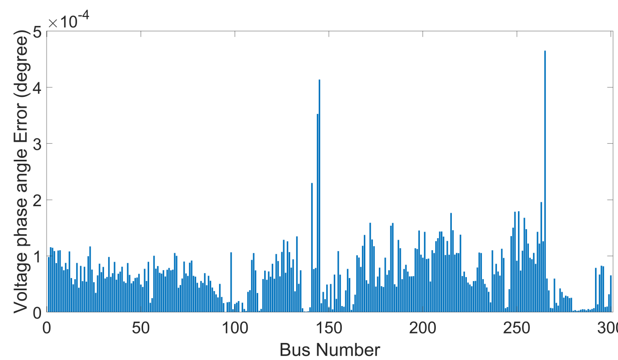

Fig. 7. Phase angles absolute error in degree for IEEE 300-bus system.

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