

Simulation of Modular Multilevel Converter System Via an Analytical Approach

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Abstract—Conventional numerical integration based simulation methods for modular multilevel converters (MMCs) need small integration time-steps to ensure simulation accuracy, which leads to a huge computation burden and limits the simulation speed. An analytical approach is proposed in this paper to reduce the computation burden and improve the time performance. Between any two adjacent switching moments, the MMC can be modeled by linear ordinary differential equations which can be quickly solved via eigenvalue decomposition techniques. A single phase 3-level MMC is used to test the proposed analytical approach and compare it with the Euler forward method in terms of simulation accuracy and computation time. The result shows that the analytical approach can greatly reduce the computation burden and simulation time without much trade-off in simulation accuracy.

Index Terms—Analytical approach, eigenvalue decomposition, linear ordinary differential equations (ODEs), modular multilevel converter (MMC)

I. INTRODUCTION

Simulation of power electronics-based systems has gained more importance in transmission level studies due to the widespread utility applications of renewable energy systems, distributed generations, and high-voltage direct current (HVDC) devices. These utility applications rely on utility-scale power electronics devices to get connected to the transmission system. Such power electronics devices will introduce much faster dynamics to the transmission system than conventional electromechanical dynamics of synchronous generators and motors. An efficient simulation tool will benefit the analysis and design of transmission systems with a high penetration of power electronics devices.

This paper mainly focuses on the simulation of modular multilevel converters (MMCs), which are widely used, e.g., HVDC transmission systems, renewable energy systems, battery energy storage system (BESS), large motor drives etc. [1]. An MMC consists of several submodules (SMs) connected in series as illustrated in Fig. 1, and produces AC voltage and current outputs by changing the inserting/bypass status of the SMs, which is referred to as switching operations in this paper. Existing simulation methods for MMC can be

roughly divided into two categories, namely numerical integration based and equivalent model based. In a numerical integration based method, a small integration time-step, e.g. several microseconds, is essential to properly simulate the switching cycle dynamics with acceptable accuracy [2], [3], e.g. the stiffness issue caused by the co-existence of fast decay and slow dynamics as later explained in section II [4]. However, the use of small time-steps is at the cost of an intensive computation burden and a slow computational speed. Therefore, many researchers resort to hardware-in-the-loop (HIL) simulation to cope with the computation burden and reach the goal of real-time simulation. HIL simulation is normally implemented in field-programmable gate arrays (FPGAs) [2][3][5] or digital signal processor (DSPs) [6]. Many researchers have also proposed numerous simplified and computationally efficient nodal equations based methods for MMCs to meet the need of fast calculation of real-time simulations [7]-[12].

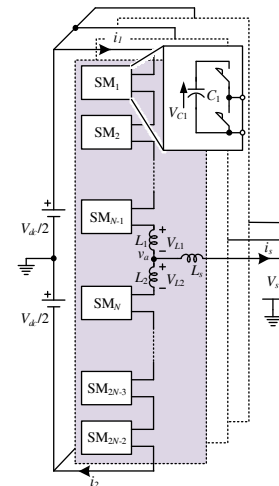


Figure 1. The topology of a three-phase MMC.

Numerical integration based methods are computationally intensive in nature due to the need of a small time-step. Another way to accelerate the simulation speed is to use a new simulation approach which can allow a larger simulation step length and thus directly reduce the computation burden. This

is motivated by the fact that the inserting/bypass status of the submodules (SMs) is fixed between any two adjacent switching moments, and the MMC system can be expressed in the form of linear ordinary differential equations (ODEs). Theoretically speaking, the simulation speed would be greatly accelerated if the linear ODEs can be solved efficiently with the given initial value, which is also the target of this paper.

In this paper, an analytical approach is investigated to efficiently find the analytical solution of the linear ODEs of the MMC system. Between any two adjacent switching moments, the analytical solution of the linear ODEs is solved using eigenvalue decomposition techniques by one-time computation, instead of the numerical solution that needs multiple small time-step integrations. This approach can benefit the converter design and converter internal control strategies, in which the simulation of switching details is widely adopted. A relatively low-cost and fast simulation platform is always preferred for analysis and performance validation. Moreover, with the simulation being accelerated, it would be more feasible to incorporate MMC into stability analysis and control of transmission level system studies [13], [14].

In the rest of this paper, the state space model of MMC system is introduced in section II. An eigenvalue decomposition based analytical approach is given in section III. The advantage of the analytical approach over conventional numerical integration methods is also discussed at the end of that section. In section IV, a single phase 3-level MMC system is used for case studies to verify the computational efficiency of the proposed method by comparing it with the Euler forward method. The conclusions and future works are discussed in section V.

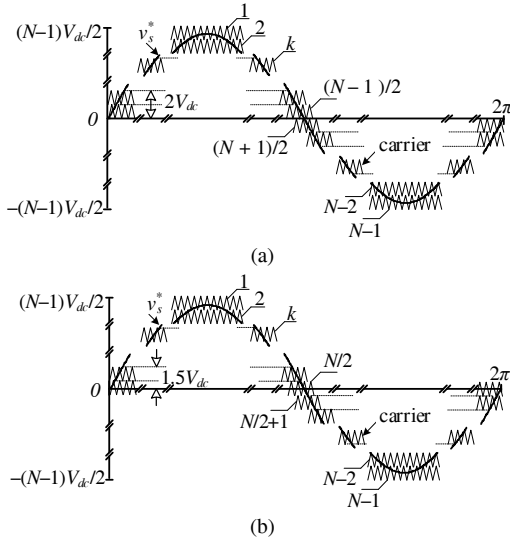


Figure 2. Ac-side voltage v_s^* and carriers, when (a) N is an odd number; and (b) N is an even number.

II. STATE-SPACE MODEL OF MMC SYSTEM

For an N -level MMC in Fig. 1, there are $N - 1$ SMs on upper arm and $N - 1$ SMs on lower arm. During normal operation, $N - 1$ out of $2N - 2$ SMs are at inserting mode, while the remaining $N - 1$ SMs are at bypass mode. Therefore,

there are N possible levels on pole voltage v_a . The pole voltage v_a follows level-shifted modulation (LSM). As shown in Fig. 2, $N - 1$ carriers determines the N -level shape of v_a . The relationship of output ac voltage reference v_s^* and $N - 1$ carriers are plotted in Fig. 2 [15]. The carriers are compared with the reference to generate the corresponding level command. The modulation strategy determines a certain submodule pattern to achieve the corresponding level. The submodule pattern can be formulated as a vector $\mathbf{y}^{(N)}$:

$$\mathbf{y}^{(N)} = [y_1 \ y_2 \ \dots \ y_{2N-2}] \quad (1)$$

where y_i is the inserting/bypass state of SM_i , $y_i = 1$ represents the inserting mode and $y_i = 0$ represents the bypass mode.

A generalized state space model can be expressed by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (2)$$

where \mathbf{x} includes all the current and voltage state variables; \mathbf{u} is a vector containing the external variables. \mathbf{A} is a matrix containing the circuit parameters and related to the inserting/bypass state vector $\mathbf{y}^{(N)}$, and \mathbf{B} only depends on the circuit parameters. Use an MMC with $N = 2$ as an example which has only two SMs, say SM_1 and SM_2 . Eq. (2) is expressed by (3) with $\mathbf{x} = [i_1, i_2, v_{C1}, v_{C2}]^T$. The coefficients marked in red contain the information from switching pattern $\mathbf{y}^{(2)}$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R}{2L} & -\frac{R}{2L} & y_1 \cdot (-\frac{1}{2L}) & y_2 \cdot (-\frac{1}{2L}) \\ -\frac{R}{2L} & -\frac{R}{2L} & y_1 \cdot (-\frac{1}{2L}) & y_2 \cdot (-\frac{1}{2L}) \\ y_1 \cdot (\frac{1}{C_1}) & 0 & 0 & 0 \\ 0 & y_2 \cdot (\frac{1}{C_2}) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{2L} & \frac{1}{2} \\ \frac{1}{2L} & -\frac{1}{2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ \frac{di_s}{dt} \end{bmatrix} \quad (3)$$

During the time interval between two adjacent switching moments, the inserting/bypass state of each SM has been determined, and there is no need to consider the capacitor voltage states of the bypassed SMs which will stay unchanged. Hence, (2) can be reduced to (4).

$$\dot{\mathbf{x}}_y = \mathbf{A}_y \mathbf{x}_y + \mathbf{B}_y \mathbf{u} \quad (4)$$

where \mathbf{x}_y contains all the states to be considered, i.e. arm circuit states and capacitor voltage states of inserting SMs. \mathbf{A}_y and \mathbf{B}_y are the resulting reduced matrices. Also, note that the external variables in \mathbf{u} can be viewed as constant during the time interval. Hence, during each time interval, the MMC model is essentially a set of linear ODEs. An example is given by using (4) and assuming SM_1 is inserted and SM_2 is bypassed, i.e. $y_1 = 1$ and $y_2 = 0$. Eq. (4) is reduced to (5) with $\mathbf{x}_y = [i_1, i_2, v_{C1}]^T$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{2L} & -\frac{R}{2L} & -\frac{1}{2L} \\ -\frac{R}{2L} & -\frac{R}{2L} & -\frac{1}{2L} \\ \frac{1}{C_1} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2L} & \frac{1}{2} \\ \frac{1}{2L} & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ \frac{di_s}{dt} \end{bmatrix} \quad (5)$$

The aforementioned stiffness issue can be characterized by the eigenvalues of \mathbf{A}_y in (4), i.e. some eigenvalues are much

smaller than others. Those small eigenvalues correspond to the fast decay dynamics, which limits the time-step length of numerical integration type methods to ensure simulation accuracy. For instance, in case studies, one of \mathbf{A}_y has two eigenvalues with quite different magnitudes, -8.65×10^5 and $6.44 \times 10^{-11} + 1.93 \times 10^{-3}i$, which shows the stiffness issue.

III. ANALYTICAL APPROACH BASED SIMULATION

A. Simulation Framework

The simulation framework for the proposed analytical approach is illustrated in Fig. 3. Overall, the simulation is still implemented in a step-by-step manner much like the conventional numerical integration methods. However, the time-step can be the same as the interval between two adjacent switching moments, which is much longer than traditional numerical integration time-step. Although only the state variables' values at the switching moments are determined, it is still adequate in terms of utility-scale application.

The PWM control block is to determine the inserting/bypass status of the SMS, and determine the next switching moment. The simulation time-step, ΔT , is also determined. Without loss of generality, in this paper, ΔT is equal to the interval between the current and the next switching moment. Hence, the state variables are only determined at the switching moments during the simulation.

The initialization block is to prepare (4) to be considered, and the initial values of \mathbf{x}_y and \mathbf{u} , say $\mathbf{x}_{y,0}$ and \mathbf{u}_0 , respectively. The initial values will be used for solving the analytical solution of (4).

The analytical solution block is to find the analytical solution of (4), which will be introduced in the next section.

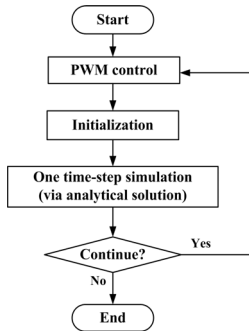


Figure 3. Simulation framework for analytical approach

B. Eigenvalue Decomposition and Analytical Solution

Simulating the ODEs in (4) is mathematically equivalent solving the analytical solution with the given initial value $\mathbf{x}_{y,0}$ and the fixed $\mathbf{u} = \mathbf{u}_0$ at $t = 0$. \mathbf{u} can be fixed since the variation in \mathbf{u} can be ignored between the adjacent switching moments.

Apply eigenvalue decomposition on \mathbf{A}_y to obtain (6).

$$\mathbf{A}_y = \mathbf{R}\mathbf{\Lambda}\mathbf{L} \quad (6)$$

where $\mathbf{\Lambda} = \text{diag}\{\lambda_i\}$ is a diagonal eigenvalue matrix; \mathbf{R} and \mathbf{L} are the right and left eigenvector matrices, respectively, and $\mathbf{R} = \mathbf{L}^{-1}$.

Replace $\mathbf{x}_y = \mathbf{R}\mathbf{z}$ into (4) to obtain (7).

$$\dot{\mathbf{z}} = \mathbf{\Lambda}\mathbf{z} + \mathbf{w} \quad (7)$$

where \mathbf{z} contains the newly transformed state variables and has the same dimension as \mathbf{x}_y ; $\mathbf{w} = \mathbf{L}\mathbf{B}_y\mathbf{u}_0$. The initial value of \mathbf{z} is computed by

$$\mathbf{z}_0 = \mathbf{L}\mathbf{x}_{y,0} \quad (8)$$

Note that the equations in (7) are actually decoupled, and each of them can be expressed in the form of (9), of which the analytical solution is easy to solve.

$$\dot{z}_i = \lambda_i z_i + w_i \quad (9)$$

In the transformed space of \mathbf{z} , the initial value is obtained by $\mathbf{z}_0 = \mathbf{L}\mathbf{x}_{y,0}$. The analytical solution of each variable z_i can be obtained from (9) as follows.

- If $\lambda_i \neq 0$, the analytical solution is

$$z_i(t) = \left(z_{i0} + \frac{w_i}{\lambda_i} \right) e^{\lambda_i t} - \frac{w_i}{\lambda_i} \quad (10)$$

- If $\lambda_i = 0$, the analytical solution is

$$z_i(t) = w_i t \quad (11)$$

After $\mathbf{z}(t)$ is solved, the analytical solution of \mathbf{x}_y can be obtained by

$$\mathbf{x}_y(t) = \mathbf{R}\mathbf{z}(t) \quad (12)$$

Then, substitute the time-step $t = \Delta T$ into (12) to obtain the state value at the next switching moment.

Note that \mathbf{A}_y and \mathbf{B}_y only depend on the SMs in inserting mode. Hence, \mathbf{R} , \mathbf{L} and λ_i can be prepared before simulation instead of during simulation, which will increase the computation efficiency. When the inserted SMs are determined, the analytical solution can be calculated using the corresponding \mathbf{R} , \mathbf{L} , λ_i and \mathbf{B}_y , the initial value $\mathbf{x}_{y,0}$ and \mathbf{u}_0 , and the time-step ΔT .

C. Discussion

In the conventional numerical integration methods, the interval between two adjacent switching moments could consist of hundreds of small simulation time-steps. For instance, if the carrier wave frequency is 1 kHz, the interval will vary between 0 μs and 1000 μs (assuming no over-modulation), and a 1000 μs interval will contain two hundred 5 μs time-step integrations. Meanwhile, the analytical approach only requires one-time computation of (8), (10), (11) and (12), which greatly reduces the computation burden. Moreover, since the analytical solution is theoretically accurate, the state variables' values at the next switching moment can be directly obtained by $\mathbf{x}_y(\Delta T)$. Hence, the proposed analytical approach is more efficient than the numerical integration methods.

From the utility-scale application point of view, the benefit of this analytical approach is two-fold. First, in terms of real-time simulation which is usually implemented on HIL simulation platform, the analytical approach could reduce the investment on the hardware side due to less computation burden. Second, when the high-performance hardware environment is not available, e.g. only CPU is accessible, the

analytical approach would use much less computation time than the numerical integration methods.

IV. CASE STUDIES

A. Case Description

A single phase 5-level MMC is utilized to test the performance of the analytical approach and compared with that of the Euler forward method. The simulation platform for programming and testing is MATLAB 2019a, on a personal computer with 3.40 GHz quad core CPU, 16 GB RAM and 64-bit system.

The single phase 5-level MMC has similar structure as Fig. 1, with only one phase being considered, and each arm consists of four submodules. Ideal switches, inductors, and capacitors with no parasitic parameters as well as ideal voltage sources are assumed in the model. The load to be served is a constant impedance load. The key parameters of the system are summarized in Table I. The switching pattern of this modeling follows the modulation discussed in Section II and [15], with the modulation index (MI) equal to 0.9. Any controller delays are not considered in the model or during simulation. The state space model can be derived following the example of (3) with $\mathbf{x} = [i_1, i_2, v_{C1}, v_{C2}, v_{C3}, v_{C4}, v_{C5}, v_{C6}, v_{C7}, v_{C8}]^T$.

TABLE I. FIVE-LEVEL MMC SIMULATION KEY PARAMETERS

Fundamental Frequency, f_0	60 Hz
Switching Frequency, f_{sw}	20 kHz
DC-Bus Voltage, V_{dc}	4000 V
Load Resistance, R_{load}	12.4 Ω
Line Inductance, L_{line}	0.1 mH
Arm Inductance, L_{arm}	0.1 μ H
Stray Resistance, R_{stray}	0.1 Ω
Submodule Capacitance, C_i	256 μ F
Number of Submodules per Arm	4

where $i = 1, 2, \dots, \text{and } 8$.

B. Comparison on Simulation Accuracy

The simulation accuracy is compared between the proposed analytical approach and Euler forward method. Euler forward method is implemented for simulation by iterating (13) over a small time-step ΔT . Eq. (13) is obtained by discretizing (2).

$$\mathbf{x}(k) = \mathbf{x}(k-1) + \mathbf{A}(k-1) \cdot \mathbf{x}(k-1) \cdot \Delta T + \mathbf{B} \cdot \mathbf{u}(k-1) \cdot \Delta T \quad (13)$$

Two different time-steps are selected for Euler forward method, including $\Delta T = 0.1 \mu\text{s}$ and $0.5 \mu\text{s}$. Euler forward method simulation with those two time-steps are referred to as EM1 and EM2, respectively. The result from EM1 is viewed to be accurate and is used as a benchmark, while EM2 is used for comparison purpose. The time-step for the analytical approach is not fixed and is determined by the interval between two adjacent switching moments. The total simulation duration is $500 \mu\text{s}$.

The simulation results are shown in Fig. 4. Note that for the analytical approach, the values of \mathbf{x} are only calculated at

the switching moments, and the dash line is drawn here only for the purposed of connecting these discrete values, not representing true dynamics. Overall the result from analytical approach (red circle) is more aligned with the result from EM1 (green curve). Fig. 5 shows a zoom-in figure for the time interval $[250, 400] (\mu\text{s})$, from which the difference between the results of EM1 and EM2 can be easily observed, while the result from analytical approach are more aligned with that of EM1.

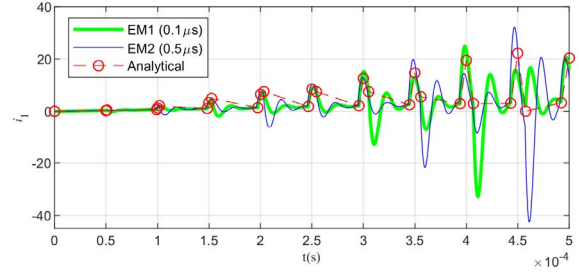


Figure 4. Comparison of simulation accuracy

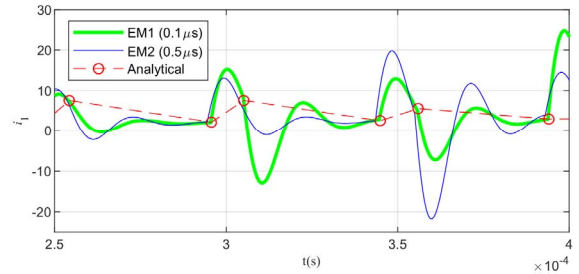


Figure 5. Zoom-in figure on interval $[250, 400] (\mu\text{s})$

The total amount of time-steps for simulating $500 \mu\text{s}$ by EM1, EM2 and analytical approach are compared in TABLE II. The analytical approach needs much less number of time-steps compared to that of Euler forward method. It verifies the advantage of the analytical approach, i.e. the computation burden is greatly reduced due to less number of total time-steps, without much trade-off on simulation accuracy.

TABLE II. TOTAL TIME-STEPS

Simulation Method	EM1	EM2	Analytical Approach
Time-steps #	5000	1000	20

C. Comparison on Computational Time

Real-time simulation is not always required especially when the hardware for simulation is not a high-performance one, like the hardware environment used in this paper. In this case, the proposed analytical approach still has certain advantage over the Euler forward method in terms of reducing computation time. This can be verified for a long simulation duration by EM1, EM2, and the proposed analytical approach, and recording their computation time.

The simulation duration is selected to be 1 s, 10 s, 20 s and 30 s. Since the computation time might also depend on the

efficiency of the simulation program, both the computation time of (13) and the whole simulation process are recorded for the Euler forward method EM1 and EM2. The computation time of analytical approach considers all the processes in Fig. 3.

TABLE III. COMPARISON OF SIMULATION TIME

Simulation Method	Computational Time			
	1 s	10 s	20 s	30 s
EM1	5.743 s (87.69 s)	60.42 s (933.61 s)	- (>1000 s)	- (>1000 s)
EM2	1.296 s (11.29 s)	13.15 s (156.68 s)	19.52 s (301.12 s)	32.48 s (464.74 s)
Analytical Approach	0.88 s	7.70 s	14.68 s	22.37 s

Computation time is recorded in TABLE III. For EM1 and EM2, the first value is the computation time for (13) and the value within the parentheses is the overall computation time. Analytical approach only considers overall computation time. The results show that analytical approach is the fastest one compared to EM1 and EM2, and it can even reach the need of real-time simulation. EM2 is much faster than EM1 but at the cost of low accuracy as aforementioned. The computation time for (13) appears to be a bottleneck for accelerating the Euler forward method. For instance, even inaccuracy issue can be improved for EM2 and the overall simulation time can be reduced due to more suitable programming language and better programming skills, it is still difficult for EM2 to reach the need of real-time simulation.

V. CONCLUSIONS AND FUTURE WORK

An analytical approach is proposed for the simulation of MMC systems, which focuses on reducing the computation burden and thus accelerate the simulation speed. A simulation framework for analytical approach is provided, and during each simulation time-step the analytical solution of the linear ODEs is obtained via eigenvalue decomposition. The proposed approach is shown to be more efficient than the Euler forward method in case studies. Moreover, from utility-scale application point of view, it would require less investment on hardware due to less computation burden in terms of real-time application, or less computation time when high-performance hardware environment is not available.

In the future, the analytical approach will be investigated for three-phase large-scale MMC systems with more SMs and

more complex control algorithms. How to incorporate MMC system simulation with the transmission system study via the analytical approach will also be investigated.

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