A General State Estimation Formulation for Three-Phase Unbalanced Power Systems

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***Abstract*—Almost all of the three-phase state estimation algorithms assume existence of a reference bus whose phase angles are perfectly balanced. This assumption is quite realistic for transmission systems, and also for most distribution systems that are connected to a strong transmission system where transmission side can modeled by a balanced reference bus. However, for distribution systems having high penetration of renewable sources or for microgrids operating in islanded mode, the assumption of a balanced reference bus will not be realistic. While there are recent publications focusing on this problem, formulation of the three-phase unbalanced state estimation problem with proper treatment of the reference bus remains unaddressed. In this paper, a new formulation will be described where an accurate state estimation solution can be obtained for any unbalanced three-phase system irrespective of its operating conditions (balanced or highly unbalanced), configuration (isolated microgrid, connected to transmission system, etc.) and whether or not it contains any synchronous generators. Validation of the proposed formulation will be carried out via simulations.**

***Index Terms*—State Estimation, Distribution System, Unbalanced Three Phase Systems, Reference Bus Phase Angle**

I. Introduction

The problem of distribution system state estimation is not new and several different algorithms developed for this purpose can be found in the literature [1]–[3]. Almost all of the developed and published methods assume existence of a reference bus whose voltage is perfectly balanced. While this assumption is not all that unrealistic since most distribution networks are connected to a strong transmission system where the transmission side can be modeled by such a balanced reference bus, it may not be the case for isolated micro-grid operation or for highly unbalanced distribution systems. Properly formulating the three-phase unbalanced state estimation problem remained unaddressed until recently. Some recent papers recognized the problem and provided alternative formulations [4]–[8]. These papers recognize that selecting a perfectly balanced voltage reference bus may not always be realistic or possible. In [4] the authors investigate the use of a single phase (say phase A) as the angle reference, and other two angular references are included as state variables. However, this choice may not always lead to a unique solution due to the rank deficiency of the measurement jacobian. The authors of [5] specify angular reference depending on the operating conditions of the system where short circuit levels of the networks are employed in unbalanced operating systems. In [6], it is stated that the entire estimation is prone to the voltage imbalance at the reference bus, and a method based on numerical observability analysis is proposed to determine the feasibility of estimating the voltage phase angles at the reference bus. This remains an ad-hoc approach without any proof of generality. An alternative solution based on the assumption of existence of a synchronous generator in the system is given in [7], where the field (excitation) voltage and synchronous reactance of the generator is used to augment the network model, allowing the use of field voltage as the balanced reference. Similarly, the Thevenin equivalent behind the root bus connected to the transmission system can be used instead as shown in [8]. Other alternatives are proposed [9] where the phase angles of the two phases are arbitrarily assigned to initialize the iterations. None of these alternatives fully address the problem as will be illustrated in the sequel. In this paper, a new formulation will be described where an accurate state estimation solution can be obtained for any unbalanced three-phase system irrespective of its operating conditions (balanced or highly unbalanced), configuration (isolated microgrid, connected to transmission system, etc.) and whether or not it contains any synchronous generators.

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II. The Proposed Method

As mentioned in the introduction section, there is no formal solution for accurate three phase state estimation when none of the buses have balanced phase angles, i.e., $\theta^A = \alpha^\circ, \theta^B = \alpha - 120^\circ, \theta^C = \alpha + 120^\circ$. The proposed method is capable of estimating unbalanced phase angles in all buses even in the absence of any synchronized phasor measurement. The philosophy behind the method will be explained as follows.

Consider the three-phase network operating under normal conditions given in the bubble in Fig. 1. The estimated bus power injection at bus r is denoted as $\bar{S}_r$. An ideal three-phase voltage source (bus v) is connected to bus r with a symmetric three-phase impedance in such a way that this connection does not change the operating state of the system, i.e., bus voltages of all buses will remain the same after the connection. The

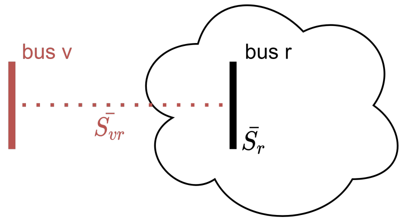

Fig. 1. Illustration of virtual bus connection

power flow from bus v to bus r is dependent to three variables as shown below:

$$\bar{S_{vr}} = f(\bar{V_v}, \bar{V_r}, Y_{vr}) \tag{1}$$

where Y_{vr} is 3×3 primitive bus admittance matrix for the line between bus v and bus k, $\bar{V_v}$ and $\bar{V_r}$ are the complex bus voltages at bus v and bus r, respectively.

Since bus v and branch v - r are fictional elements, and hypothetically connected to the physical power network, bus v and branch v-r wll be referred as virtual bus and virtual branch throughout the manuscript. The three-phase model of the virtual bus connection is given in Fig. 2. As the virtual bus is assumed to be ideal voltage source, the voltage magnitudes will be unknown but assumed to be the same for all phases, i.e., $V_0^a = V_0^b = V_0^c = V_0$. Moreover, the virtual bus voltage phase angles will be displaced by 120°. Using the three-phase model and variables given in Fig. 2, equation 1 can be re-arranged as shown in (2).

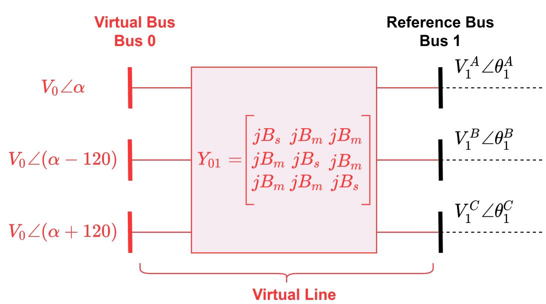

Fig. 2. Three-phase model of reference bus when virtual bus is connected

$$\begin{bmatrix} P_{10}^A \\ P_{10}^B \\ P_{10}^C \\ Q_{10}^A \\ Q_{10}^B \\ Q_{10}^C \end{bmatrix} = f(V_0, \alpha, V_1^A, V_1^B, V_1^C, \theta_1^A, \theta_1^B, \theta_1^C, B_s, B_m) \tag{2}$$

There are 6 measurements, and 7 unknowns in the problem formulation given in (2) considering voltage magnitudes of reference bus are estimated/known. The problem can be reformulated by arbitrarily setting $\alpha = 0°$, and the other two phase angles of virtual bus as $-120°$ & $120°$. This will not change the solution except for the phase angles of all the buses which will all shift by α. Thus, it can be concluded that the problem in (2) can be solved with six measurements equations and six unknowns which implies that it should be possible to estimate the voltage magnitude at virtual bus, susceptance values of virtual line, and unbalanced phase angles of reference bus given enough measurement redundancy in the original system.

Therefore, the proposed method enables extension of any given power network by using six additional measurements and six newly introduced system states, hence facilitating the accurate estimation of the unbalanced phase angles of all buses in the power system. The main advantage of employing virtual bus will be the possibility of using a bus with balanced phase angles even though none of the original system buses are balanced, hence none can be used as the reference bus.

The proposed method is based on the following rules in formulating the estimation problem:

- The bus voltage magnitude at each phase of the virtual bus is equal to each other. $V_0^A = V_0^B = V_0^C = V_0$.
- The phase angles of virtual bus voltage are displaced by 120 degrees from each other, i.e., $\theta_0^A = \alpha°, \theta_0^B = \alpha - 120°, \theta_0^C = \alpha + 120°$
- There is enough measurement redundancy at the reference bus to provide system observability in the proposed model. Note that this is not a restriction since there should be at least one bus with power injection and voltage magnitude measurements.
- The virtual branch that connects the virtual bus to the system is purely reactive, i.e., $Y_{01} = jB_{01}$. Moreover, the admittance matrix of the corresponding line is symmetrical and composed of two unknown variables as given below.

$$Y_{01} = \begin{bmatrix} jB_s & jB_m & jB_m \\ jB_m & jB_s & jB_m \\ jB_m & jB_m & jB_s \end{bmatrix}$$

State estimation formulation, measurement configuration and structure of the measurement jacobian for the proposed formulation will be explained in the following sub-sections.

A. State Estimation

The relation of the system states and measurements for the power system SE is given in (3) . In this SE formulation, three phase model of the network is used in order to account for unbalanced operation and non-symmetrical line geometry.

$$z = h(x) + \varepsilon \tag{3}$$

where z is the $(m \times 1)$ measurement vector, x is the $(n_s \times 1)$ state vector, ε is the $(m \times 1)$ measurement error vector, $h(.)$ is the nonlinear function relating states to measurements, n_s is the number of system states, m is the number of measurements. Measurement error vector, ε, is unknown but assumed to have zero mean Gaussian distribution [10]. The Least Absolute Value (LAV) based state estimation problem is formulated as:

$$\min \sum_{i=1}^{m} |r_i| \tag{4}$$

$$\text{subject to} \quad z_i = h_i(x) + r_i \quad i = 1, 2, ...m$$

where, r_i is the residual of i^{th} measurement, i.e., $r_i = z_i - h(\hat{x}_i)$. Using the first-order approximation of $h_i(x^0)$ around of x^0, the problem can be rewritten as a sequence of linear optimization problem as:

$$\min \sum_{i=1}^{m} |r_i| \qquad (5)$$
$$\text{subject to} \quad \Delta z = H \cdot \Delta x$$

Typically, state vector, x, is composed of bus voltage magnitudes and phase angles. The proposed method introduces three new variables, namely bus voltage magnitude of the virtual bus (V_0), self (B_s) and mutual (B_m) susceptances of the virtual branch, in addition to the conventional system states. The structure of the modified state vector will be as follows:

$$x^T = [\underbrace{B_s, B_m, V_0}_{(1\times3)}, \underbrace{\theta_1^{ABC}, \ldots, \theta_n^{ABC}}_{(1\times3n)}, \underbrace{V_1^{ABC}, \ldots, V_n^{ABC}}_{(1\times3n)}] \qquad (6)$$

In a network with n buses, the size of the state vector will be $(6n+3)\times 1$ while 3 of the states are associated with virtual elements, $3n$ of the states are three phase voltage magnitudes, and $3n$ of the states are three phase phase angles.

B. Measurement Configuration

Extending the power system model to connect the proposed virtual branch and bus, the measurement set will have to be adjusted accordingly. This is accomplished by converting the power injection measurement into an equivalent branch flow measurement along the virtual branch from the reference bus towards the virtual bus.

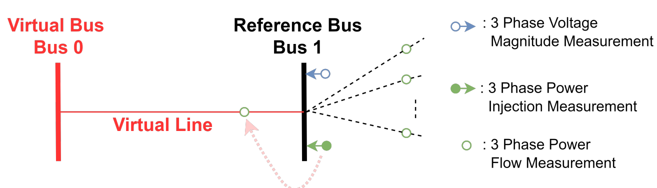

Fig. 3. Measurement configuration of reference bus when virtual bus is connected

As explained above, the virtual bus is connected to the network in such a way that this connection will not change the operating state of the system. In other words, the power flow from virtual bus to reference bus should be equal to the net power injection at the reference bus. Using this information, hypothetical virtual flow measurements ($P_{10}^{ABC}, Q_{10}^{ABC}$) are introduced to the measurement set. In Fig. 3, the physical power network is indicated in black, and virtual elements are shown in red. The flow measurement along the virtual branch ($P_{10}^{ABC}, Q_{10}^{ABC}$) is given in (7), and the updated injection measurement at the reference bus (P_1^{*ABC}, Q_1^{*ABC}) is shown in (8). Note that zero injection information at the reference bus is not critical for the SE; however, it is required to detect and identify possible bad data in the measurement set.

$$P_{10}^{ABC} = -P_1^{ABC}, \quad Q_{10}^{ABC} = -Q_1^{ABC} \qquad (7)$$
$$P_1^{*ABC} = 0, \quad Q_1^{*ABC} = 0 \qquad (8)$$

C. Measurement Jacobian

The elements of the measurement jacobian, H, are obtained by taking the partial derivative of measurement functions with respect to system states. Unlike the conventional formulation, the structure of the H matrix is slightly different since a limited number of measurements are function of newly introduced states, B_s, B_m *and* V_0. $P_{10}^{ABC}, Q_{10}^{ABC}$ and P_1^{ABC}, Q_1^{ABC} are functions of B_s, B_m *and* V_0. Thus, the first three columns of H will contain non-zeros only in the corresponding rows of virtual branch flow measurement and the reference bus injection measurement assuming that the first three columns are partial derivatives with respect to B_s, B_m *and* V_0.

The structure of the H matrix is shown in (9) where the column size of the Jacobian is $(6n+3)\times 1$.

$$\mathbf{H} = \begin{array}{c} \begin{matrix} m\times1 & m\times1 & m\times1 & m\times3n & m\times3n \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\partial V_i^{ABC}}{\partial V} \\ \frac{\partial P_{10}^{ABC}}{\partial B_s} & \frac{\partial P_{10}^{ABC}}{\partial B_m} & \frac{\partial P_{10}^{ABC}}{\partial V_0} & \frac{\partial P_{ij}^{ABC}}{\partial \theta} & \frac{\partial P_{ij}^{ABC}}{\partial V} \\ \frac{\partial Q_{10}^{ABC}}{\partial B_s} & \frac{\partial Q_{10}^{ABC}}{\partial B_m} & \frac{\partial Q_{10}^{ABC}}{\partial V_0} & \frac{\partial Q_{ij}^{ABC}}{\partial \theta} & \frac{\partial Q_{ij}^{ABC}}{\partial V} \\ \frac{\partial P_1^{*ABC}}{\partial B_s} & \frac{\partial P_1^{*ABC}}{\partial B_m} & \frac{\partial P_1^{*ABC}}{\partial V_0} & \frac{\partial P_j^{ABC}}{\partial \theta} & \frac{\partial P_j^{ABC}}{\partial V} \\ \frac{\partial Q_1^{*ABC}}{\partial B_s} & \frac{\partial Q_1^{*ABC}}{\partial B_m} & \frac{\partial Q_1^{*ABC}}{\partial V_0} & \frac{\partial Q_j^{ABC}}{\partial \theta} & \frac{\partial Q_j^{ABC}}{\partial V} \end{bmatrix} \end{array} \qquad (9)$$

where;

V_i^{ABC} : three-phase voltage magnitude at Bus i where $i \in [1, n]$

$P_{10}^{ABC}, Q_{10}^{ABC}$: three-phase real/reactive flow measurement at virtual line from reference bus to virtual bus

$P_{ij}^{ABC}, Q_{ij}^{ABC}$: three-phase real/reactive flow measurement from Bus i to Bus j where $j \in [1, n]$

P_1^{ABC}, Q_1^{ABC}: three-phase real/reactive power injection measurement at the reference bus

P_i^{ABC}, Q_i^{ABC}: three-phase real/reactive power injection measurement at Bus i

III. Tutorial Example

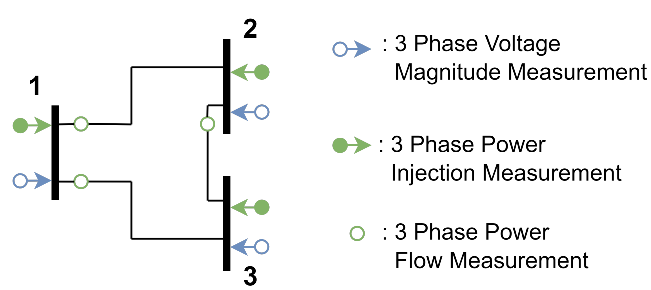

Fig. 4. Sample three bus system with measurement configuration

A 3-bus system shown in Fig. 4 will be used to illustrate the solution procedure of the proposed method. To highlight the capability of the proposed method in handling heavily unbalanced systems, the sample system is simulated with highly unbalanced loading. The normal operating state of the system is given in Table I where the voltage phase angles of the reference bus (bus 1) are clearly not balanced.

TABLE I
NORMAL OPERATING STATE VOLTAGE SOLUTION OF THE SAMPLE 3 BUS SYSTEM

	Phase Angle (degree)			Voltage Magnitude (pu)		
# of Bus	A	B	C	A	B	C
1	-4.939	-118.069	119.742	1.062	1.085	0.990
2	-5.761	-118.004	119.894	1.052	1.089	0.982
3	-5.398	-117.969	119.649	1.058	1.086	0.982

As apparent from the table, there are no buses with balanced voltage phase angles that can be selected as a reference bus for state estimation. Conventional SE models either utilize balanced reference bus phase angles which cause inaccuracy or assign only one phase angle of the reference bus, and estimate other two phases which may create convergence issues or lead to multiple solutions. On the other hand, the proposed method remains completely robust against such issues and yields an accurate state estimation solution.

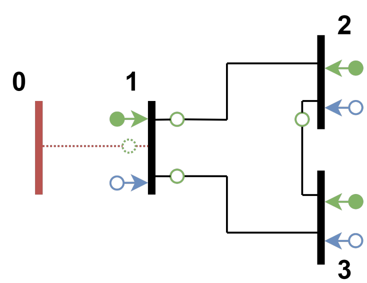

Fig. 5. Sample three bus system with hypothetical virtual bus connection

As described above, a virtual bus with voltage magnitude $V_0^A = V_0^B = V_0^C = V_0$, and phase angle $\theta_0^A = 0°, \theta_0^B = -120°, \theta_0^C = 120°$ is connected to Bus 1 via a virtual branch as shown in Fig. 5. The (3×3) primitive bus admittance matrix of the virtual branch is given in (10) where B_s and B_m are the self and mutual susceptances respectively. Once the virtual bus is hypothetically connected to the network, the state vector, x, is formed as shown in (11).

$$Y_{01} = \begin{bmatrix} jB_s & jB_m & jB_m \\ jB_m & jB_s & jB_m \\ jB_m & jB_m & jB_s \end{bmatrix} \tag{10}$$

$$x = [B_s, B_m, V_0, \theta_1^{ABC}, \theta_2^{ABC}, \theta_3^{ABC}, V_1^{ABC}, V_2^{ABC}, V_3^{ABC}] \tag{11}$$

The conventional states are initialized at the flat start values, i.e., $V^{ABC} = 1pu$, $\theta^A = 0°, \theta^B = -120°, \theta^C = 120°$. The initial value of B_s can be assigned a value between 0 and -1000, and B_m can be assigned a value between 0 and 100. While the algorithm consistently converges irrespective of the existing imbalance, the number of required iterations for convergence will vary based on the chosen initial values of B_s and B_m. However, during online operation, once the solution is obtained for one measurement scan, estimated values of B_s and B_m from the previous estimation cycle can be used as initial values which drastically minimizes the number of iterations. Furthermore, the original measurements received from SCADA which are denoted by Z_{org} are modified once the hypothetical virtual bus is connected. Additional measurements, P_{10}^{ABC} and Q_{10}^{ABC} are calculated using (7), and zero injections are used at bus 1 as given in (8). The resulting modified measurement set, Z_{mod}, is shown in (12) where the modified measurements are highlighted in red.

$$Z_{org} = \begin{bmatrix} V_1^{ABC} \\ V_2^{ABC} \\ V_3^{ABC} \\ P_{12}^{ABC}/Q_{12}^{ABC} \\ P_{13}^{ABC}/Q_{13}^{ABC} \\ P_{23}^{ABC}/Q_{23}^{ABC} \\ P_1^{ABC}/Q_1^{ABC} \\ P_2^{ABC}/Q_2^{ABC} \\ P_3^{ABC}/Q_3^{ABC} \end{bmatrix}_{45\times1} \quad Z_{mod} = \begin{bmatrix} V_1^{ABC} \\ V_2^{ABC} \\ V_3^{ABC} \\ P_{10}^{ABC}/Q_{10}^{ABC} \\ P_{12}^{ABC}/Q_{12}^{ABC} \\ P_{13}^{ABC}/Q_{13}^{ABC} \\ P_{23}^{ABC}/Q_{23}^{ABC} \\ P_1^{*ABC}/Q_1^{*ABC} \\ P_2^{ABC}/Q_2^{ABC} \\ P_3^{ABC}/Q_3^{ABC} \end{bmatrix}_{51\times1} \tag{12}$$

The state estimation is carried out using the modified measurement set and the state vector. At the end of each iteration, bus admittance matrix, Y_{bus}, is updated with the estimated B_s, B_m values. Once the estimator converges, the variables associated with the virtual bus and virtual branch can be neglected, since they are fictional variables with no practical use. To sum up, the proposed estimator is capable of estimating three phase unbalanced phase angles at all buses even when there is no bus with balanced phase angles to be used as a reference bus.

IV. SIMULATIONS

To validate and evaluate the proposed method, results for 3 bus system and IEEE 14 bus system are given in this section. The accuracy of the proposed method, which is labeled as Method I throughout results section, is compared with the performance of the methods available in the literature. Method II is the conventional approach where angular references are assigned balanced phase angles, i.e., $\theta_1^A = 0°, \theta_1^B = -120°, \theta_1^A = 120°$, and corresponding columns of reference bus phase angles in Jacobian matrix are removed. Lastly, in Method III, only one phase (Phase A) of reference bus is assigned $\theta_1^A = 0°$, and corresponding column is removed from Jacobian matrix. Phase B and C of reference bus phase angles are included as system states, and estimated using the measurements.

The accuracy of the estimator is evaluated using Mean Square Error (MSE) metric. The formulation of the MSE metric is given in (13). The additional states, V_0, B_s and B_m, are fictional variables with no true values, and their values do not carry any physical meaning. Hence, they are ignored while the MSE is calculated.

$$MSE = \sqrt{\frac{1}{n_{cs}} \cdot (X_{estimated} - X_{true})^2} \tag{13}$$

where n_{cs} is the number of conventional states, $X_{estimated}$ is the estimated conventional states, and X_{true} is the true conventional

TABLE II
3 BUS SYSTEM WITH BASE CASE LOADING

Bus		True Values		Method I		Method II			Method III-A		Method III-B	
		V(pu)	$\theta(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}$ $(^\circ)$	$\hat{\theta}_{rotated}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$
1	A	1.062	-4.939	1.062	-4.939	1.061	0	-4.939	1.062	-4.939	1.062	-4.938
	B	1.085	-118.069	1.085	-118.069	1.084	-120	-124.939	1.085	-118.347	1.090	62.224
	C	0.990	119.742	0.990	119.742	0.990	120	115.061	0.990	119.761	0.989	133.926
2	A	1.052	-5.761	1.052	-5.762	1.052	-0.832	-5.771	1.052	-5.761	1.052	-5.774
	B	1.089	-118.004	1.089	-118.004	1.089	-119.896	-124.834	1.089	-118.280	1.083	62.145
	C	0.982	119.894	0.982	119.894	0.981	120.131	115.193	0.981	119.913	0.981	134.153
3	A	1.058	-5.398	1.058	-5.399	1.057	-0.473	-5.412	1.058	-5.398	1.057	-5.413
	B	1.086	-117.969	1.086	-117.969	1.086	-119.878	-124.816	1.086	-118.246	1.086	62.121
	C	0.982	119.649	0.982	119.649	0.981	119.895	114.956	0.981	119.668	0.981	133.899

Estimates of Virtual Elements	$\hat{B}_s = -237.581pu$	$\hat{B}_m = 89.216pu$	$\hat{V}_0 = 1.050pu$

TABLE III
3 BUS SYSTEM WITH MODIFIED LOADING

Bus		True Values		Method I		Method II			Method III-A		Method III-B	
		V(pu)	$\theta(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}$ $(^\circ)$	$\hat{\theta}_{rotated}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$	$\hat{V}(pu)$	$\hat{\theta}(^\circ)$
1	A	1.063	-6.479	1.063	-6.504	1.061	0	-6.479	1.063	-6.479	1.0629	-6.47
	B	1.118	-118.019	1.118	-118.021	1.084	-120	-126.479	1.118	-118.019	1.1179	-478.11
	C	0.991	121.122	0.991	121.105	0.990	120	113.521	0.991	121.121	-0.990	-58.85
2	A	1.045	-7.966	1.045	-8.027	1.044	-1.506	-7.985	1.045	-7.966	1.0453	-7.96
	B	1.013	-117.970	1.013	-117.972	1.094	-119.865	-126.344	1.128	-117.971	1.1274	-1198.06
	C	0.979	121.556	0.979	121.540	0.977	120.379	113.900	0.979	121.556	0.9795	-238.42
3	A	1.055	-7.261	1.055	-7.323	1.053	-0.805	-7.284	1.054	-7.261	1.0548	-7.26
	B	1.122	-117.930	1.122	-117.932	1.088	-119.863	-126.342	1.122	-117.931	-1.121	-658.02
	C	0.981	121.169	0.981	121.153	0.979	120.017	113.538	0.981	121.170	0.9814	481.19

Estimates of Virtual Elements	$\hat{B}_s = -330.28pu$	$\hat{B}_m = 125.79pu$	$\hat{V}_0 = 1.061pu$

states. Conventional states are the bus voltage magnitudes and phase angles.

A. 3 Bus System

The 3 bus system given in tutorial example is used to demonstrate the performance of the proposed method. The measurement configuration is shown in Fig. 5. To highlight the superiority of the proposed formulation, the accuracy of the estimator is compared with two other methods described above. The normal operating state of the system is obtained using three phase power flow solver. System loading is intentionally made heavily unbalanced to highlight the performance improvement. Furthermore, in addition to the base case loading scenario, a modified loading case is employed to demonstrate the varying virtual element estimates to adapt the changing operating points. The estimator converges in 8 iterations in both loading cases.

Table II and III show the true voltage solution obtained by power flow solution, and estimation estimates by three different methods. As given in tables, once the loading of the system is modified, the estimates of virtual elements also change to satisfy the operating conditions. The proposed method (Method I) yields highly satisfactory results with $MSE < 10^{-10}$ in both loading cases as given in Table IV. When Method II is utilized, where the reference bus is assumed to have balanced phase angles, the MSE of the estimator is calculated as $MSE = 0.0035$ for the base case loading, and $MSE = 0.0067$ for slightly more unbalanced modified loading case. Note that while MSE is calculated using Method II, $\hat{\theta}_{rotated}$ is utilized. $\hat{\theta}_{rotated}$ is obtained by shifting all phase angles by true phase angle of θ_1^A since not the angles but angle differences matter.

TABLE IV
MSE OF SE ALGORITHMS IN 3 BUS SYSTEM USING DIFFERENT METHODS

	MSE for Base Case	MSE for Modified Loading
Method I	3.49×10^{-11}	6.91×10^{-11}
Method II	0.0035	0.0067
Method III-A	6.349×10^{-6}	5.45×10^{-6}
Method III-B	1.6585	32.306

In the the case of Method III, the solution is found to be highly sensitive to the initialization of the angular references. While initial values of the reference bus phase angles are set to be $\theta_1^A = 0^\circ$, $\theta_1^B = -121^\circ$ and $\theta_1^C = 121^\circ$ in III-A; they are set to be $\theta_1^A = 0^\circ$, $\theta_1^B = -125^\circ$ and $\theta_1^C = 121^\circ$ in III-B. Although estimator works properly in III-A with reasonable MSE as shown in Table IV, it converges to completely different infeasible voltage solution due to initialization of state variables in III-B. Hence, it can be concluded that specifying only one angular reference (Method III) yields multiple solutions which prevents method to be reliable.

The results obtained using 3 bus system show that accuracy of the estimation is significantly improved with the proposed method. Method II may be accurate enough to be utilized in the practical distribution systems; however, its performance in highly unbalanced systems like this ones is questionable. On the other hand, Method III highly rely on initialization of the state variable since it has multiple convergence points. Hence, utilization of Method III is not trivial due to the lack of a formal initialization methodology.

B. IEEE 14 Bus System

The performance of the SE using Method I/II/III in IEEE 14 bus system are also given, and evaluated. Moreover, estimates of virtual elements under two different loading scenarios are shown in Table V. Although the estimates of virtual components are not meaningful alone, they allow unbalanced angular references to be estimated by adopting to the varying loading. A similar fashion to 3 bus case is observed in IEEE 14 bus system results. MSE of the estimation is significantly improved using the proposed method compared to Method II as shown in Table VII in both loading scenarios. Considering the modified loading is slightly more unbalanced, the accuracy of Method II decreases compared to base case as shown in VII. Thus, the superiority of the proposed method over Method II becomes more evident as the system becomes more unbalanced. The accuracy of the Method III is reasonable when the proper initial values are assigned to Phase B and C of reference bus phase angles; however, as illustrated in case III-B, an infeasible solution is obtained with different initialization of state variables which makes the method unreliable overall.

TABLE V
ESTIMATES OF VIRTUAL ELEMENTS UNDER VARYING LOADING

	$\hat{B}_s$	$\hat{B}_m$	$\hat{V}_0$
Base Case	$-27.746pu$	$2.211pu$	$1.349pu$
Modified Loading	$-57.321pu$	$42.100pu$	$1.452pu$

TABLE VI
ESTIMATE OF REFERENCE BUS PHASE ANGLES IN IEEE 14 BUS SYSTEM BY VARIOUS METHODS

	True Values	Method I	Method II	Method III-A	Method III-B
$\hat{\theta}_1^A(°)$	-1.708	-1.708	-1.708	-1.708	-1.708
$\hat{\theta}_1^B(°)$	-124.029	-124.030	-121.708	124.028	-42.527
$\hat{\theta}_1^C(°)$	119.570	119.569	118.2920	119.559	272.155

where reference bus phase angles are initialized as $\theta_1^A = 0°$, $\theta_1^B = -121°$ and $\theta_1^C = 121°$ in III-A, and $\theta_1^A = 0°$, $\theta_1^B = -115°$ and $\theta_1^C = 125°$ in III-B.

TABLE VII
MSE OF SE ALGORITHMS IN IEEE 14 BUS SYSTEM USING DIFFERENT METHODS

	MSE for Base Case	*MSE* for Modified Loading
Method I	6.69×10^{-11}	1.23×10^{-10}
Method II	3.042×10^{-4}	0.0007
Method III-A	2.89×10^{-8}	6.78×10^{-8}
Method III-B	13.42	8.749

V. CONCLUSION

This paper presents a new method to obtain accurate state estimation solution for any three-phase unbalanced power system irrespective of its operating condition or configuration. The novel contribution of this study is a proper treatment of angular reference where there is no balanced bus to be assigned as reference bus. The method employs coupled three-phase network model, and propose an accurate state estimation formulation for power systems.

It is shown that the developed estimator is capable of converging in each case and estimating accurately in various networks with different characteristics. The simulation results prove the superiority of the proposed method in terms of accuracy of estimation. Although the improvement is not clearly apparent in the mildly unbalanced cases, the performance differs significantly under highly unbalanced or islanded operating scenarios.

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