Improved Line Outage Detection in Transmission Systems with Few PMUs

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***Abstract*—Detection of line outages in large power grids with a limited number of PMUs remains a challenging problem. Previous solutions proposed for this problem relied on sparse estimation methods and used the DC network model. While they were shown to be effective for most line outage cases, they occasionally failed due to the badly conditioned network matrices. In this paper, the sparse estimation formulation is further improved to handle such badly conditioned cases, thus significantly increasing the reliability and effectiveness of the approach for different network configurations.**

***Index Terms*—QR Decomposition, Sparse Estimation, Line Outage, LASSO, Outage Detection**

I. Introduction

Reliable operation of a power system depends on accurate monitoring and control of the operating state of the system. Monitoring is commonly accomplished via a state estimator which not only provides the best estimate of the system state but also detects and removes bad measurements. In doing all this, the state estimator implicitly assumes that the network model is perfectly known. This assumption may not always hold true. A line may be disconnected but this information may somehow not be reported to the control center. In such cases, results of the state estimator will be biased and these results also impact other application functions such as the contingency analysis, optimal power flow as well as market functions. Thus, it is crucial to detect changes in the network topology in a timely manner and to prevent such errors to create risky operating conditions leading to blackouts as witnessed earlier in [1]–[3].

The problem of tracking network topology and specifically line outages is worked on and proposed solutions are presented in the literature [4]–[6]. These approaches are well developed and described as real-time applications, however they are not widely used due to their scalability limitations based on the use of graph search algorithms and implementation complexity. An alternative line outage detection approach uses sparse estimation and is based on the well documented Least Absolute Shrinkage and Selection Operator (LASSO). It relies on measurements received from only a few PMUs in the transmission system [7], [8]. The sparse estimation problem is formulated using the DC network model, that is given in compact form as ($B\Delta\theta = \Delta P$) and used by the LASSO solution [7], [8]. Although the DC network model provides a good approximation for various network applications of transmission systems, it does not lend itself equally well when using the inverse formulation, i.e. expressing the phase angles in terms of the bus power injections. This is the case in LASSO based sparse estimation, where several columns of B^{-1} may be co-linear. In such cases, LASSO could misidentify or fail to identify the correct outage [9], [10].

In this paper, the line outage detection problem formulation is modified in order to avoid or significantly minimize the co-linearity related deficiency of the method while still using the DC network model in LASSO algorithm. The main objective is to eliminate the co-linearity issue in sparse estimation so that the line outage detection is highly accurate. This is accomplished by using the "QR" decomposition to transform the coefficient matrix which significantly improves its condition number and minimizes the collinearities in the formulation of sparse estimation problem. [11].

The contributions of the proposed method are listed below:

- The proposed modification enables line outage detection with fewer PMUs,
- Implementation of the proposed method is easy and requires only a minor modification to the original sparse estimation problem,
- The proposed method decreases the co-linearity of coefficient matrix for sparse estimation,
- The proposed method improves the accuracy of line outage detection without significant additional computational cost.

The paper is organized as follows; in Section II, the DC power flow model is reviewed. In Section III, the problem formulation is explained in detail. In Section IV, the formulation of the sparse estimation problem for the proposed line outage detection is described. Section V provides results of simulations to illustrate the proposed method's performance for example systems. Section VI concludes the paper.

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II. DC Power Flow Model

Consider the net active power injection at bus k expressed as [12]:

$$P_k = \sum_{j=1}^{N} = |V_k||V_j|(G_{kj}cos(\theta_k - \theta_j) + B_{kj}sin(\theta_k - \theta_j)) \quad (1)$$

where,

- $|V_k|$ is the voltage magnitude of bus k,
- $|V_j|$ is the voltage magnitude of bus j,
- G_{kj} is the real part of branch admittance value between bus k and bus j,
- B_{kj} is the imaginary part of branch admittance value between bus k and bus j,
- θ_k is the voltage angle at bus k,
- θ_j is the voltage angle at bus j.

The following assumptions generally hold true for transmission systems [13]:

- Transmission line R/X" ratios are low, typically between 0.1 and 0.5,
- Phase angle differences between neighboring buses are small, typically less then 10 degrees,
- Bus voltage magnitudes do not deviate significantly from 1.0 p.u.

The above assumptions can be used to simplify (1) as:

$$P_k = \sum_{j=1}^{N} = B_{kj}(\theta_k - \theta_j) \quad (2)$$

Writing the above in compact matrix form:

$$B\theta = P \quad (3)$$

where,

- P is the active power injection vector,
- θ is the voltage angle vector,
- B is the imaginary part of bus admittance matrix.

III. Problem Formulation

DC power flow model is commonly used in several network applications since it provides a reasonably accurate linear approximation between real power injections and voltage phase angles [13]. Line outage detection problem is one of those applications that lend itself well to the linear formulation relating the changes in bus phase angles ($\Delta\theta$) to the changes in real power injections (ΔP) [14]:

$$B\Delta\theta = \Delta P \quad (4)$$

where,

- $B = \begin{cases} \text{if} \quad k \neq j \rightarrow B_{kj} = -\frac{1}{x_{kj}} \\ \text{if} \quad k = j \rightarrow B_{kk} = \sum_{j=1, j\neq k}^{N} \frac{1}{x_{kj}} \\ 0 \quad otherwise \end{cases}$

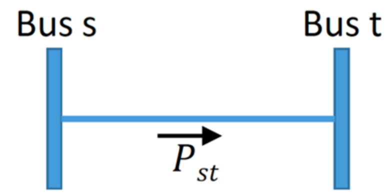

Fig. 1a: Pre-contingency flow on branch s-t.

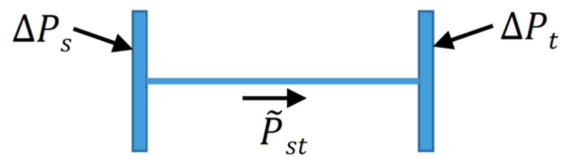

Fig. 1b: Post-contingency flow on branch s-t.

- x_{kj} is the branch reactance value between bus k and bus j,
- $\Delta\theta$ is the vector of phase angle differences between post outage theta and pre outage conditions,
- ΔP is the vector of bus injection changes between post and pre outage conditions. Assuming that loads and generation remains unchanged during the outage event, the only nonzeroes are expected at the disconnected line terminal buses.

Let us partition the system buses into two sets, buses with and without PMU measurements using the subscripts i and e for buses with and without PMU measurements respectively. The partitioned form of (4) will be written as:

$$\begin{bmatrix} \Delta P_e \\ \Delta P_i \end{bmatrix} = \begin{bmatrix} B_{ee} & B_{ei} \\ B_{ie} & B_{ii} \end{bmatrix} \begin{bmatrix} \Delta\theta_e \\ \Delta\theta_i \end{bmatrix} + \begin{bmatrix} \Delta e_e \\ \Delta e_i \end{bmatrix} \quad (5)$$

where,

- subscript i refers to buses with PMUs and,
- subscript e refers to buses without PMUs.

Consider disconnection of a single line which will result in changes in B and $\Delta\theta$ to reflect this topology change. It is possible to keep B unchanged and model the line outage by virtual power injections (ΔP_s and ΔP_t) at the terminal buses of the disconnected branch. This equivalent representation of the line outage by using virtual power injections can be illustrated using Fig. 1a and 1b.

The virtual power injections (ΔP_s and ΔP_t) in Fig. 1b have the same magnitude but opposite signs, and their absolute values are equal to the real power flow of the line, making the net flow effectively zero while yielding the terminal bus phase angles of the post-outage solution. This simple equivalent facilitates modeling of outages without physically removing the lines, but modifying the terminal power injections as:

$$\Delta P_s = -\Delta P_t = \tilde{P_{st}} \quad (6)$$

IV. Proposed Method

In order to solve (5), a well-documented and used sparse solution method, namely, LASSO can be used [15]. Using the partitioned equation, changes in phase angles can be expressed as:

$$\begin{bmatrix}\Delta\theta_e\\ \Delta\theta_i\end{bmatrix} = \begin{bmatrix}B_{ee} & B_{ei}\\ B_{ie} & B_{ii}\end{bmatrix}^{-1} \begin{bmatrix}\Delta P_e\\ \Delta P_i\end{bmatrix} + \begin{bmatrix}\Delta e_e\\ \Delta e_i\end{bmatrix} \tag{7}$$

In (7) B^{-1} is typically quite badly conditioned, i.e. it is nearly singular. Hence, application of LASSO algorithm becomes numerically difficult due to the difficulty to properly identify a linearly independent set of columns. So, instead of using B^{-1} directly, it can be first transformed by a QR decomposition. Note that, the application assumes availability of PMU measurements only at a limited number of system buses. Therefore, applying the QR decomposition without partitioning B as in (7) will not be possible unless all buses are equipped with PMUs allowing calculation of the product $R\Delta\theta$. Based on the above considerations, the following ordered formulation is used instead:

$$Q_{ordered}R_{ordered}\Delta\theta_{ordered} = \Delta P_{ordered} \tag{8}$$

$$\begin{bmatrix}Q_{ee} & Q_{ei}\\ Q_{ie} & Q_{ii}\end{bmatrix} \begin{bmatrix}R_{ee} & R_{ei}\\ 0 & R_{ii}\end{bmatrix} \begin{bmatrix}\Delta\theta_{ee}\\ \Delta\theta ii\end{bmatrix} = \begin{bmatrix}\Delta P_{ee}\\ \Delta P_{ii}\end{bmatrix} \tag{9}$$

$$\begin{bmatrix}R_{ee} & R_{ei}\\ 0 & R_{ii}\end{bmatrix} \begin{bmatrix}\Delta\theta_{ee}\\ \Delta\theta ii\end{bmatrix} = \begin{bmatrix}Q_{ee}^T & Q_{ie}^T\\ Q_{ei}^T & Q_{ii}^T\end{bmatrix} \begin{bmatrix}\Delta P_{ee}\\ \Delta P_{ii}\end{bmatrix} \tag{10}$$

$$R_{ii}\Delta\theta_{ii} = [Q_{ei}^T\ Q_{ii}^T] \begin{bmatrix}\Delta P_{ee}\\ \Delta P_{ii}\end{bmatrix} \tag{11}$$

By solving (11) using LASSO, the virtual power injections at the terminal buses of disconnected lines can be obtained. In addition to that, the number of non-zeros found by LASSO solution can be reduced further by utilizing the connectivity matrix A. This will be accomplished by the following modified formulation:

$$R_{ii}\Delta\theta_{ii} = \begin{bmatrix}{Q^T}_{ei} & {Q^T}_{ii}\end{bmatrix} A^T_{ordered}\Delta\widetilde{P} \tag{12}$$

where,

- $\Delta\widetilde{P}$ is the virtual branch power flow vector,
- $\Delta P = A^T * \Delta\widetilde{P}$ is the virtual power injection vector,
- $A_{ordered}$ is the connectivity matrix $b \times n$ with columns corresponding to locations of with PMUs and without PMUs.
- n is the number of buses and,
- b is the number of branches in the system.

Connectivity matrix A of (12) is defined as:

$$A\left(k,m\right) = \begin{cases} 1 & \text{if } k \text{ is the sending bus at branch } m\\ -1 & \text{if } k \text{ is the receiving bus at branch } m\\ 0 & \text{otherwise}\end{cases} \tag{13}$$

where, $m = 1, ..., b$.

After incorporating the connectivity matrix as in (12), the optimization problem to be solved by LASSO will take the following form:

$$\Delta\tilde{P} := \min_{\Delta\widetilde{P}} ||\Delta\theta' - M\Delta\widetilde{P}||_2^2 + \lambda||\Delta\widetilde{P}||_1 \tag{14}$$

where,

- $\Delta P = A^T\Delta\widetilde{P}$,
- $M = \begin{bmatrix}{Q^T}_{ei} & {Q^T}_{ii}\end{bmatrix} A^T$,
- $\Delta\theta' = R_{ii}\Delta\theta_{ii}$.

While the above LASSO formulation using the partitioned QR decomposition significantly improves the solution, there will still be cases where the virtual power injections identified by $\Delta\widetilde{P}$ vector may not correspond to the true outage, especially when there are too few PMUs installed in the system. To identify and eliminate such false line outage detections during the sparse estimation process, the post-outage power flow given in $\Delta\widetilde{P}$ is compared with the predicted $\widetilde{P}_{st}$ value which is given by:

$$\widetilde{P}_{st} = \frac{P_{st}}{1 - PTDF_{(s,t),s,t}} \tag{15}$$

where,

- P_{st} refers to pre-outage flow along the line connecting bus s and bus t,
- $\widetilde{P}_{st}$ refers to virtual power flow which models the outage without physically removing the line.

$$PTDF_{(s,t),s,t} = \frac{1}{x_{st}}[(X_{ss} - X_{st}) - (X_{st} - X_{tt})] \tag{16}$$

where,

- s and t refers to sending and receiving terminals of the disconnected line,
- $X_{ss},\ X_{tt},\ X_{st}$ refer to B^{-1} matrix entries corresponding to the line s-t,
- x_{st} refers to reactance of the branch between bus s and bus t.

If the mismatch between the result given by LASSO algorithm and $\widetilde{P}_{st}$ evaluated by (15) exceeds a specified margin, then the corresponding row of the detected line outage in A will be set to zero. This process is repeated until the mismatch satisfies the margin which is selected as 15%, since there may be a value of λ that will make the LASSO result more accurate.

The line outage detection algorithm is summarized as follows:

Step 1 Form B matrix, $\Delta\theta$ vector and connectivity matrix.
Step 2 Reorder B matrix and $\Delta\theta$ vector with respect to PMU locations.
Step 3 Apply Eq. 9, Eq. 10, Eq. 11 and Eq. 12 to find M and $\Delta\theta'$.
Step 4 Apply LASSO method in Eq. 14 to find non-zero values.

Step 5 Calculate $\widetilde{P}_{st}$ value according to Eq. 15 and Eq. 16. If the two values match in determined margin, the outage is successfully detected, and algorithm terminates. Otherwise go to Step 6.
Step 6 Set corresponding row of the A matrix to zero and go back to Step 4.

The aim of the proposed method is to decrease the co-linearity of the impedance matrix by utilizing QR decomposition, and obtaining more accurate results at the end of sparse estimation process.

V. Simulation Results

The proposed line outage detection method is tested on two different systems, namely the IEEE 118 bus system shown in Fig. 2 and the IEEE 300 bus system for various line outage scenarios [16]. It is assumed that a small number of synchronized voltage phasor measurements provided by PMUs are available in both systems.

A. IEEE 118 Bus System

The locations of these limited number of PMUs for IEEE 118 bus system are selected as buses 2, 13, 22, 39, 49, 53, 58, 63, 81, 84, 103, 105, 106, 114 by trial and error method to provide most accurate results. Moreover, each of these PMUs are assumed to have a channel to measure also one current phasor along an incident branch. These current phasors enable calculation of the voltage phasors at the remote end of the respective branches. Hence, using these available voltage phasors and current phasors, phase angles of buses 2, 12, 13, 15, 22, 23, 39, 40, 53, 54, 58, 56, 63, 59, 81, 80, 84, 85, 103, 110, 105, 108, 106, 107, 114, 115 are calculated for the IEEE-118 bus system.

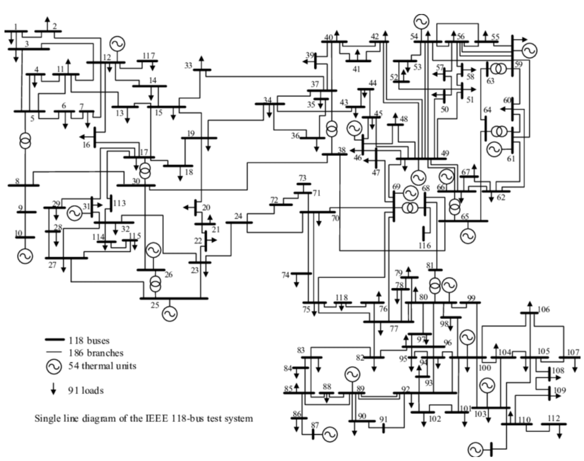

Fig. 2: IEEE 118 Bus System.

IEEE 118 bus system contains 179 lines. However, not all of these lines significantly affect the system state when disconnected. Therefore, additional sensitivity analysis is performed to detect which line outages affect the system states more than 3%. Branches with less than 3% sensitivity are disregarded from consideration.

In order to perform the sensitivity analysis, each branch is disconnected one at a time from the system. The power flow solution for the pre-outage and post-outage configurations are obtained. Using the changes in system states between pre-outage and post-outage for the corresponding disconnected branch, significant line outages are selected. The sensitivity analysis for the IEEE 118 bus system identifies 129 line outages as significant. Once the number of significant line outages is determined, that many simulation cases are created and tested using the LASSO algorithm in Matlab environment [17]. The improvements in the proposed method in terms of the decreased number of iterations for each line outage case and improvement of the success rate can be seen in Table I and Table II.

As evident in Table I, the proposed method yields significantly better results in terms of the line outage detection percentage. Note that in the conventional formulation B^{-1} is highly ill-conditioned due to the near co-linearity of its columns. This is graphically illustrated in Fig. 3. This often leads to selection of the wrong line outage by the LASSO algorithm since there may be a branch whose column is highly correlated with the column of the actually disconnected branch. The impact of the proposed approach on the co-linearity of the columns of B^{-1} can be seen in Fig. 4. As evident from Table II, the success rate of the line outage detection method is significantly increased as a result of using the proposed approach.

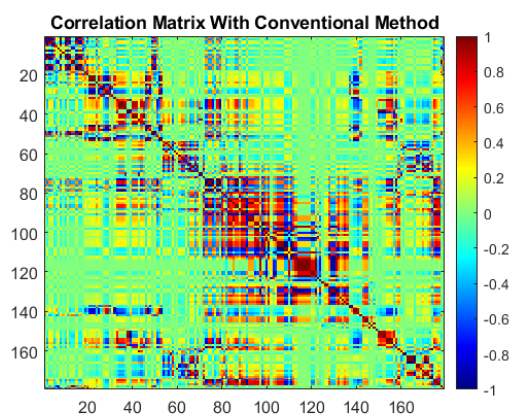

Fig. 3: Correlation Graph of Conventional B^{-1} matrix.

Considering the results shown in Table II, the conventional implementation of the sparse estimation method fails to detect 15 out of 126 line outages using only 14 PMUs. However, introducing the above described QR decomposition based modification to the sparse estimation procedure, this number can be decreased to 6 out of 126 line outages. Although the performance of the conventional method approaches to that of the proposed method when the number of PMUs is increased, this will increase the installation cost due to the additional PMUs required. Therefore, the proposed approach can provide improved success rate for line outage detection with no additional installation cost.

TABLE I: The results of line outage detection algorithm with conventional method vs proposed method with 14 PMU.

Outage Line **Branch / Phase /** $\widetilde{P_{st}}$	**Detected Outage With Conventional Method** **Final Iteration # / Branch /** $\Delta\widetilde{P}$			**Detected Outage With Proposed Method** **Final Iteration # / Branch /** $\Delta\widetilde{P}$		
42-49 / -4.4153	11	Not Detected	~	4	42-49	-4.5193
59-61 / -0.6758	11	Not Detected	~	2	58-61	-0.7177
82-83 / -4.6718	11	Not Detected	~	1	82-83	-4.8802
64-61 / 1.2346	11	Not Detected	~	1	64-61	1.1854
8-5 / 34.3989	11	Not Detected	~	1	8-5	36.9010
94-100 / 0.1305	11	Not Detected	~	2	94-100	0.1084
94-95 / 1.4166	10	82-96	-0.3570	3	94-95	1.1456
89-92 / 13.5493	11	Not Detected	~	7	89-92	10.4212
101-102 / -1.3665	1	92-90	0.4860	3	101-102	-1.3464

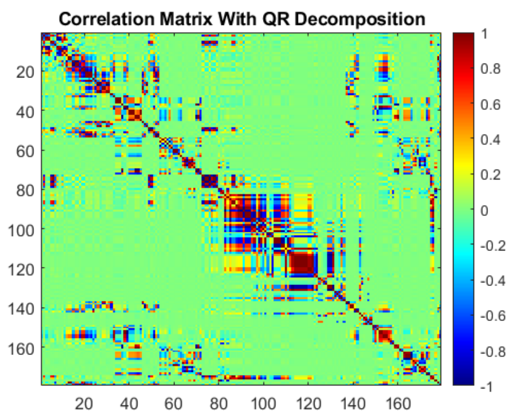

Fig. 4: Correlation Graph after Applying the Proposed QR Decomposition.

TABLE II: Success rates of conventional vs proposed line outage detection algorithm.

Method	**Significant Line Outage Number**	**Success Rate**	**PMU Number**
Conventional Method	126	88.88%	14
Conventional Method	126	89.62%	15
Conventional Method	126	95.23%	20
Conventional Method	126	96.82%	21
Proposed Method	126	95.23%	14
Proposed Method	126	96.03%	15
Proposed Method	126	97.61%	20
Proposed Method	126	98.41%	21

B. IEEE 300 Bus System With 2% Load Variation

In IEEE 118 bus system, loads were kept constant between two consecutive PMU scans. However, power systems, loads may vary between two consecutive PMU measurement scans. Hence, IEEE 300 Bus system is used to compare the conventional and proposed methods under load variations between two measurement scans. It is assumed that the load variation remains within 2% between two measurement scans and the proposed line outage method is performed to detect line outages in load variations between two consecutive PMU scans.

To obtain pre-outage and post-outage system states "Matpower" toolbox in Matlab environment is utilized [18] and initially pre-outage power flow solutions are calculated as a base case. Then, before calculating the power flow solution of post-outage system states for various line outage cases, all loads are increased by 2%, and line outage scenarios are created with respect to this 2% load variations. Among these line outage scenarios in IEEE 300 bus system, 322 out of 411 line outage cases converged. Then, to determine the significant line outages among the converged line outage cases, sensitivity analysis is performed. Sensitivity analysis selected 217 out of 322 line outages as significant, i.e. loss of these lines will modify the system states by more than 3%.

After, finding the significant line outages, PMU locations are determined as done in the IEEE 118 bus case, and 19 PMUs are placed in IEEE 300 bus system. The results of comparison between proposed method and conventional method for 2% load variations can be seen in Table III.

TABLE III: The success rate of 2% load variation case with conventional method vs proposed method.

Method	**Significant Line Outage Number**	**Success Rate**	**PMU Number**
Conventional Method	322	74.19%	19
Conventional Method	322	80.64%	21
Conventional Method	322	83.41%	23
Proposed Method	322	83.41%	19
Proposed Method	322	88.01%	21
Proposed Method	322	90.32$	23

Table III shows that using 19 PMU, 56 line outage cases will be missed by using only the conventional method allowing 2% load variations. However, this number is reduced down to 36 using the proposed method. These results imply that with increasing system size, line outage detection by the proposed method becomes more effective, especially when working with few installed PMUs.

VI. Conclusion

The objective of the proposed method described in this paper is to increase the detectability of line outages in large power systems. This is achieved by modifying the coefficient matrix of the LASSO via the application of QR transformation to the coefficient matrix. As evident from the presented simulation results, the QR decomposition feature improves the success rate of the line outage algorithm, despite the use of a limited number of PMUs in the system.

Even in relatively small size systems, proposed method is quite effective in reducing the cost of investment into new PMUs. For large scale systems it will be proportionally more significant.

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