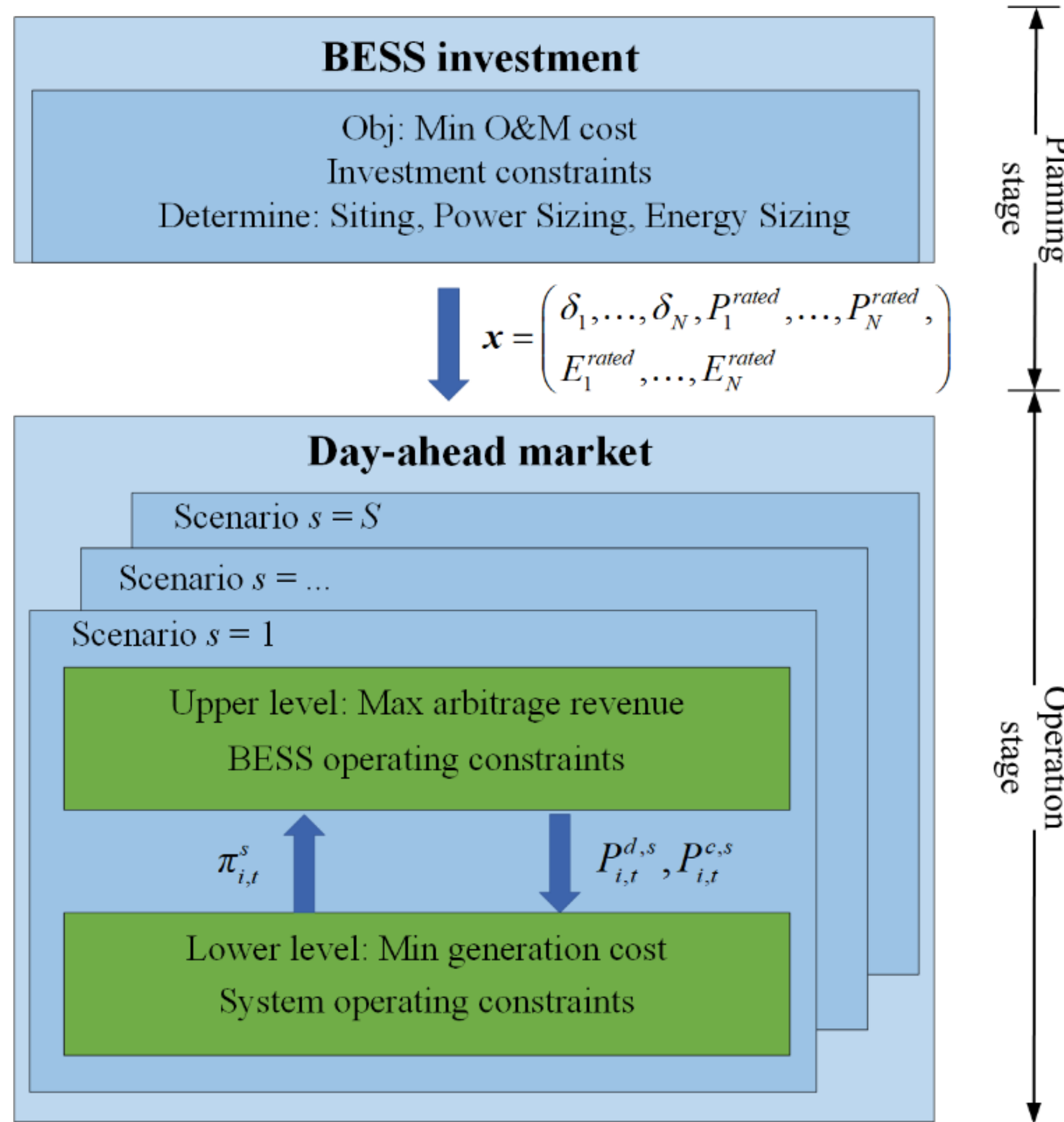


Contributions

- The DLMP is applied as the price signal to incentivize the BESS planning in a deregulated distribution system.
- A TS-SBP arbitrage model is established.
- A k -means-based scenario extraction algorithm is proposed to extract the most representative patterns of LMP and system load profiles.
- BESS candidate bus reduction and inactive voltage constraint reduction are proposed to reduce the computational complexity of this large-scale optimization problem.

Framework of the TS-SBP model



Problem Formulation

First Stage: Optimal Siting & Sizing

$$\max - \sum_{i \in \Omega_{BS}} (c^{M_i} P_i^{rated} + c^{E_i} E_i^{rated}) + E[f(\mathbf{x}, s)] \quad (1)$$

$$s.t. \quad \sum_{i \in \Omega_{BS}} \delta_i \leq N_{BS}^{max} \quad (2)$$

$$\sum_{i \in \Omega_{BS}} k^P P_i^{rated} + k^E E_i^{rated} \leq C^{Bgt} \quad (3)$$

$$P_i^{min} \delta_i \leq P_i^{rated} \leq P_i^{max} \delta_i \quad (4)$$

$$E_i^{min} \delta_i \leq E_i^{rated} \leq E_i^{max} \delta_i \quad (5)$$

$$E_i^{rated} = 4 \cdot P_i^{rated} \quad (6)$$

$$E[f(\mathbf{x}, s)] = 365 \cdot \sum_{s \in S} p(s) f(\mathbf{x}, s) \quad (7)$$

Second Stage: BESS Operation in a Deregulated Distribution Market

Upper level

$$f(\mathbf{x}, s) = \max \sum_{i \in \Omega_T} \sum_{i \in \Omega_{BS}} \pi_{i,t}^s \cdot P_{i,t}^{BESS,s} \quad (8)$$

$$= \max \sum_{i \in \Omega_T} \sum_{i \in \Omega_{BS}} \pi_{i,t}^s \cdot (\sqrt{\eta_i} P_{i,t}^{d,s} - P_{i,t}^{e,s} / \sqrt{\eta_i}) \quad (8)$$

$$s.t. \quad E_{i,t+1}^s = E_{i,t}^s + P_{i,t}^{e,s} - P_{i,t}^{d,s} \quad (9)$$

$$E_{i,t=0}^s = E_{i,t=T}^s \quad (10)$$

$$SOC_i^{min} \cdot E_i^{rated} \leq E_{i,t+1}^s \leq SOC_i^{max} \cdot E_i^{rated} \quad (11)$$

$$0 \leq P_{i,t}^{e,s} \leq P_i^{rated}, 0 \leq P_{i,t}^{d,s} \leq P_i^{rated} \quad (12)$$

Lower level

$$\min h(\mathbf{z}, \mathbf{y}, s) =$$

$$\sum_{i \in \Omega_T} \left(\sigma_{sub,t}^{p,s} P_{sub,t}^{G,s} + \sigma_{sub,t}^{q,s} \tilde{Q}_{sub,t}^{G,s} + \sum_{i \in \Omega_G} (\sigma_{i,t}^{p,s} P_{i,t}^{G,s} + \sigma_{i,t}^{q,s} \tilde{Q}_{i,t}^{G,s}) \right) \quad (13)$$

$$s.t. \quad P_{sub,t}^{G,s} + \sum_{i \in \Omega_G} P_{i,t}^{G,s} + \sum_{i \in \Omega_{BS}} P_{i,t}^{BESS,s} = \sum_{i \in \Omega_D} P_{i,t}^{D,s} + P_t^{L,s} : \lambda_t^{p,s} \quad (14)$$

$$Q_{sub,t}^{G,s} + \sum_{i \in \Omega_G} Q_{i,t}^{G,s} = \sum_{i \in \Omega_D} Q_{i,t}^{D,s} + Q_t^{L,s} : \lambda_t^{q,s} \quad (15)$$

$$V_{j,t}^s = V_{sub,t}^s + \sum_{i \in \Omega_T} Z_{ji}^p (P_{i,t}^{G,s} + P_{i,t}^{BESS,s} - P_{i,t}^{D,s}) + \sum_{i \in \Omega_G} Z_{ji}^q (Q_{i,t}^{G,s} - Q_{i,t}^{D,s}) \quad (16)$$

$$V_{j,t}^{min} \leq V_{j,t}^s \leq V_{j,t}^{max} : \omega_{j,t}^{min,s}, \omega_{j,t}^{max,s}, \forall j \in \Omega_N \quad (17)$$

$$P_{i,t}^{G,min} \leq P_{i,t}^{G,s} \leq P_{i,t}^{G,max} : \omega_{i,t}^{min,s}, \omega_{i,t}^{max,s}, \forall i \in \Omega_{MG} \quad (18)$$

$$0 \leq Q_{i,t}^{G,s} \leq P_{i,t}^{G,s} \tan(\arccos \alpha_i) : \omega_{i,t}^{q,min,s}, \omega_{i,t}^{q,max,s}, \forall i \in \Omega_{MG} \quad (19)$$

$$Q_{i,t}^{G,min} \leq Q_{i,t}^{G,s} \leq Q_{i,t}^{G,max} : \omega_{i,t}^{q,min,s}, \omega_{i,t}^{q,max,s}, \forall i \in \Omega_{SVC} \quad (20)$$

$$-Q_{i,t}^{G,s} \leq \tilde{Q}_{i,t}^{G,s}, \tilde{Q}_{i,t}^{G,s} \leq \tilde{Q}_{i,t}^{G,s} : \kappa_{i,t}^{-s}, \kappa_{i,t}^{+s}, \forall i \in \Omega_G \quad (21)$$

$$\pi_{i,t}^s = \lambda_t^{p,s} + \lambda_t^{q,s} \cdot \frac{\partial P_{i,t}^{loss,s}}{\partial P_{i,t}^{D,s}} + \lambda_t^{q,s} \cdot \frac{\partial Q_{i,t}^{loss,s}}{\partial P_{i,t}^{D,s}} \quad (22)$$

Overall Solution

Algorithm 2: Overall Solution Procedure

1. **Decomposition:** Since there is a finite set of scenarios, (25) can be reformulated as:

$$\max 365 \cdot \sum_{s \in S} p(s) (-c^T \mathbf{x}_s + \pi_s^T \mathbf{y}_s)$$

Decompose it into S subproblems.

2. **Initialization:** For each $s \in S$, compute:

$$(\mathbf{x}_s, \mathbf{y}_s) \in \arg \max -c^T \mathbf{x}_s + \pi_s^T \mathbf{y}_s$$

3. **Candidate buses reduction:** Obtain the aggregated binary variable: $\hat{\delta} = \sum_{s \in S} p(s) \delta_s$, where $\hat{\delta} = \{\hat{\delta}_1, \dots, \hat{\delta}_{\Omega_N}\}$; remove $\hat{\delta}_i$ with low values; the rest are the most probable buses.

4. **Voltage constraints reduction:** Check $V_s = \{V_{11}, \dots, V_{\Omega_N}\}$, $s \in S$, identify buses at which voltage constraints are never violated; then, remove constraints at these buses.

5. **Solving:** With reduced candidate buses and voltage constraints, compute: $(\mathbf{x}, \mathbf{y}_s) \in \arg \max -c^T \mathbf{x} + 365 \cdot \sum_{s \in S} p(s) \pi_s^T \mathbf{y}_s$.

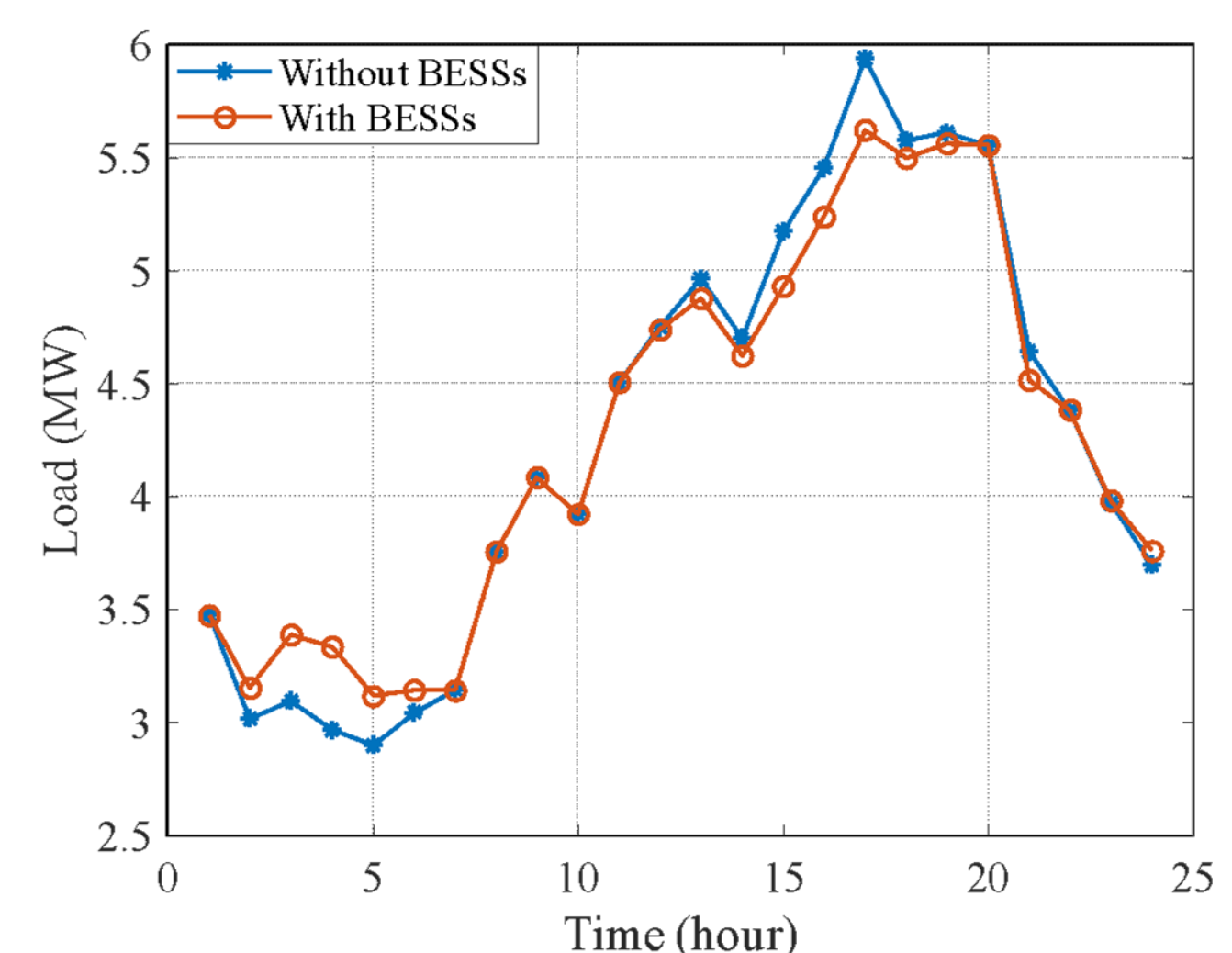
6. **Voltage constraints update:** Check whether the removed voltage constraints are violated or not. If yes, add the violated ones and go back to Step 5; otherwise, the algorithm terminates.

Simulation Results

Siting and sizing results

Cases	BESS bus (#)	Power/Energy (kW/kWh)	Annual net profit (\$)	Time (s)
Case 1	11, 15, 18, 31, 33	100/400, 149/597, 103/414, 107/426, 100/400	9302.22	7936
Case 2	11, 15, 18, 31, 33	100/400, 149/597, 103/414, 107/426, 100/400	9129.37	3442
Case 3	11, 15, 18, 31, 33	100/400, 149/597, 103/414, 107/426, 100/400	9129.38	1338
Case 4	11, 15, 18, 31, 33	100/400, 149/597, 103/414, 107/426, 100/400	9129.38	2390
Case 5	11, 15, 18, 31, 33	100/400, 149/597, 103/414, 107/426, 100/400	9129.38	1077

System load profiles with and without BESSs



Conclusions

- The DLMP can act as an effective price signal to incentivize BESS planning.
- The proposed two scale-reduction strategies are verified to improve computational efficiency and maintain accuracy.
- Optimal siting and sizing benefit both BESS investors and the DSO.

