Tri-level Hybrid Interval-Stochastic Optimal Scheduling for Flexible Residential Loads under GAN-assisted Multiple Uncertainties

Xiaofei Wang[1], Fangxing Li[1], Jin Zhao[1], Mohammed Olama[2], Jin Dong[2], Hang Shuai[1], Teja Kuruganti[2]
[1] The University of Tennessee, Knoxville
[2] Oak Ridge National Laboratory

Contributions

- Uncertainties from all three layers and multiple sources are modeled.
- A feasible hybrid interval-stochastic bilevel programming model is established to simulate the interdependence of load aggregators and the DSO.
- A rolling horizon optimization (RHO) scheme is employed to continuously optimize the consumption schedule.

Uncertainties Modeling for Residential Appliances

Single HVAC Model

$$\theta^h_{i,j,t+1} = a^h_{i,j}\theta^h_{i,j,t} + b^h_{i,j}\tilde{\theta}^{out}_t + g^h_{i,j}u^h_{i,j,t} + \tilde{\varepsilon}^h_{i,j,t}$$
$$\theta^{h,\min} \le \theta^h_{i,j,t} \le \theta^{h,\max}$$
$$\tilde{\varepsilon}^h_{i,j,t} \sim N\left(0, \left(\alpha^h \theta^{h,mid}\right)^2\right)$$

HVAC aggregator model

$$\theta^h_{i,t+1} = a^h_i \theta^h_{i,t} + b^h_i \tilde{\theta}^{out}_t + g^h_i u^h_{i,t} + \tilde{\varepsilon}^h_{i,t}$$
$$\theta^{h,\min} \le \theta^h_{i,t} \le \theta^{h,\max}$$
$$syn^{h,\min}_i \le u^h_{i,t} \le syn^{h,\max}_i$$
$$-\Delta u^{h,dr}_i \le u^h_{i,t+1} - u^h_{i,t} \le \Delta u^{h,ur}_i$$
$$SOC^h_{i,t} = \frac{\theta^{h,\max} - \theta^h_{i,t}}{\theta^{h,\max} - \theta^{h,\min}}$$
$$SOC^{h,\min}_i \le SOC^h_{i,t} \le SOC^{h,\max}_i$$
$$P^H_{i,t} = u^h_{i,t} \cdot N^h_i \cdot P^{h,rated}$$
$$\tilde{\theta}^{out}_t \in \left[\theta^{out}_t - \alpha^{temp}\theta^{out}_t, \theta^{out}_t + \alpha^{temp}\theta^{out}_t\right]$$
$$\tilde{\varepsilon}^h_{i,t} \sim N\left(0, {\sigma^h_{i,t}}^2\right)$$
$$\sigma^h_{i,t} = \sqrt{\sum_{j=1}^{N^h_i}\left(\alpha^h \theta^{mid}\right)^2} \Big/ N^h_i$$

Bi-level Formulation With Uncertainties

First level: minimize electricity bill

$$\min \sum_{t\in T}\left(\sum_{i\in H}\pi^p_{i,t}\cdot P^H_{i,t} + \sum_{i\in W}\pi^p_{i,t}\cdot P^W_{i,t} + \sum_{i\in E}\pi^p_{i,t}\cdot P^C_{i,t}\right)$$
$$s.t.$$

HVAC aggregator constraints

EWH aggregator constraints

EV aggregator constraints

Second level: market-clearing

$$\min \sum_{t\in T}\sum_{i\in G} c_{i,t}\cdot P^G_{i,t}$$
$$s.t.$$
$$\sum_{i\in G}P^G_{i,t} + \sum_{i\in PV}\tilde{P}^{PV}_{i,t} = \sum_{i\in B}P^D_{i,t} + \sum_{i\in H}P^H_{i,t} + \sum_{i\in W}P^W_{i,t} + \sum_{i\in E}P^C_{i,t} + P^{loss}_t : \lambda^p_t$$
$$\sum_{i\in G}Q^G_{i,t} = \sum_{i\in B}Q^D_{i,t} + Q^{loss}_t : \lambda^q_t$$
$$V^{\min} \le V_{sub,t} + \sum_{i\in B}Z^p_{j,i}\left(P^G_{i,t} + \tilde{P}^{PV}_{i,t} - P^D_{i,t} - P^H_{i,t} - P^W_{i,t} - P^C_{i,t}\right) +$$
$$\sum_{i\in B}Z^q_{j,i}\left(Q^G_{i,t} - Q^D_{i,t}\right) \le V^{\max} : \omega^{v,\min}_{j,t}, \omega^{v,\max}_{j,t}$$
$$P^{G,\min}_{i,t} \le P^G_{i,t} \le P^{G,\max}_{i,t} : \omega^{p,\min}_{i,t}, \omega^{p,\max}_{i,t}$$
$$Q^{G,\min}_{i,t} \le Q^G_{i,t} \le Q^{G,\max}_{i,t} : \omega^{q,\min}_{i,t}, \omega^{q,\max}_{i,t}$$
$$\pi^p_{i,t} = \lambda^p_t + \left(\lambda^p_t \cdot \frac{\partial P^{loss}_t}{\partial P^D_{i,t}} + \lambda^q_t \cdot \frac{\partial Q^{loss}_t}{\partial P^D_{i,t}}\right) + \sum_{j\in B}\left(\omega^{v,\min}_{j,t} - \omega^{v,\max}_{j,t}\right)Z^p_{j,i}$$

Uncertainty Handling

First level: interval optimization

State-space expression

$$\boldsymbol{\theta} = \boldsymbol{A}^{-1}\left(\boldsymbol{B}\tilde{\boldsymbol{\theta}}^{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \tilde{\boldsymbol{\varepsilon}}\right)$$
$$\tilde{\boldsymbol{\varepsilon}} \in \left[-2\boldsymbol{\sigma}, 2\boldsymbol{\sigma}\right]$$
$$\boldsymbol{\theta}^{\min} \le \boldsymbol{A}^{-1}\left(\boldsymbol{B}\tilde{\boldsymbol{\theta}}^{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \tilde{\boldsymbol{\varepsilon}}\right) \le \boldsymbol{\theta}^{\max}$$

Optimistic model

$$\boldsymbol{A}^{-1}\left(\boldsymbol{B}\underline{\boldsymbol{\theta}}_{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \underline{\boldsymbol{\varepsilon}}\right) \le \boldsymbol{\theta}^{\max}$$
$$\boldsymbol{\theta}^{\min} \le \boldsymbol{A}^{-1}\left(\boldsymbol{B}\overline{\boldsymbol{\theta}}_{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \overline{\boldsymbol{\varepsilon}}\right)$$

Pessimistic model

$$\boldsymbol{A}^{-1}\left(\boldsymbol{B}\overline{\boldsymbol{\theta}}_{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \overline{\boldsymbol{\varepsilon}}\right) \le \boldsymbol{\theta}^{\max}$$
$$\boldsymbol{\theta}^{\min} \le \boldsymbol{A}^{-1}\left(\boldsymbol{B}\underline{\boldsymbol{\theta}}_{out} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C} + \underline{\boldsymbol{\varepsilon}}\right)$$

Second level: stochastic optimization

PV Scenario generation

Generative Adversarial Networks (GANs)

PV Scenario reduction

Algorithm 1: Scenario Reduction

1. Initialization: Calculate the Euclidean distance between point forecast profile and all scenarios.

$$d(\overline{\boldsymbol{P}}^{PV}, \boldsymbol{P}^{PV,s}) = \left\|\overline{\boldsymbol{P}}^{PV} - \boldsymbol{P}^{PV,s}\right\|_2,\ s = 1,2,\ldots,S$$

where $\overline{\boldsymbol{P}}^{PV}$ is the point-forecast power, and $\boldsymbol{P}^{PV,s}$ is the generated scenario.

2. Candidate Scenario set: Choose the closest 30% of all scenarios as the candidate scenario set S_C, with the probability of each candidate scenario $p = 1/|S_C|$;

3. Kantorovich probability distance-based scenario reduction:

3.1 Eliminate scenario s_m if it meets the following condition.

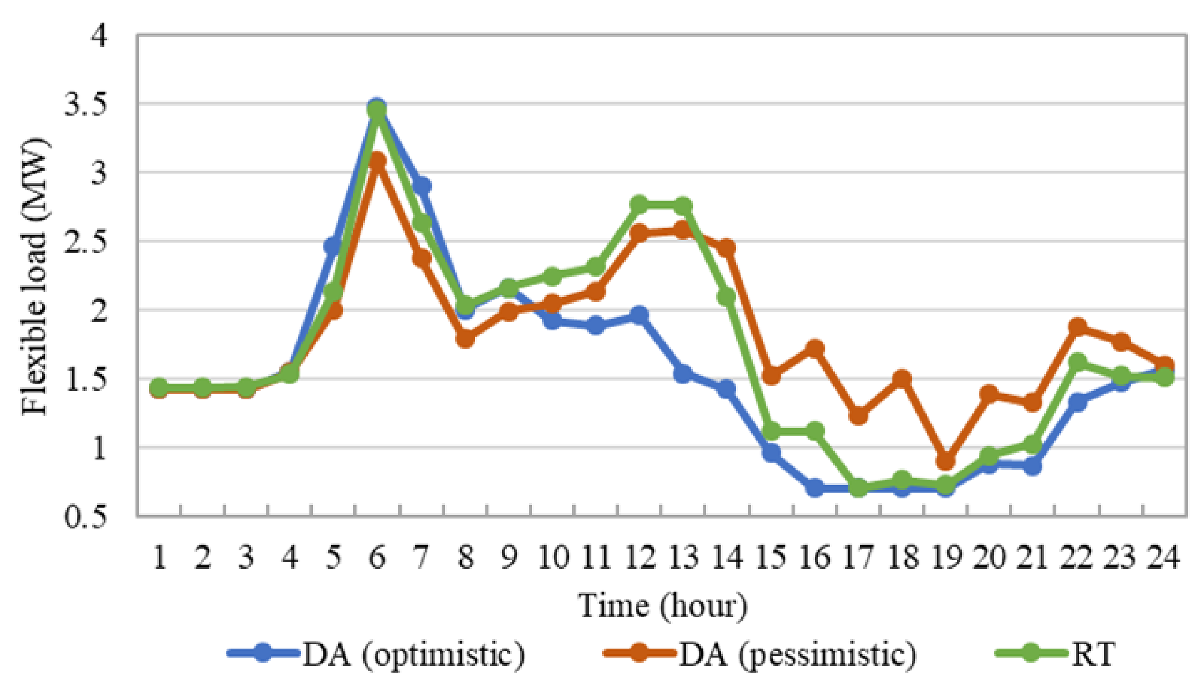
$$d_m = \min\left\{ p(m)\cdot p(n)\cdot d(\boldsymbol{P}^{PV,m}, \boldsymbol{P}^{PV,n})\right\}\ m,n \in \{1,\ldots,S_C\}, n \ne m$$

3.2 Update the probability of s_n and the number of scenarios.

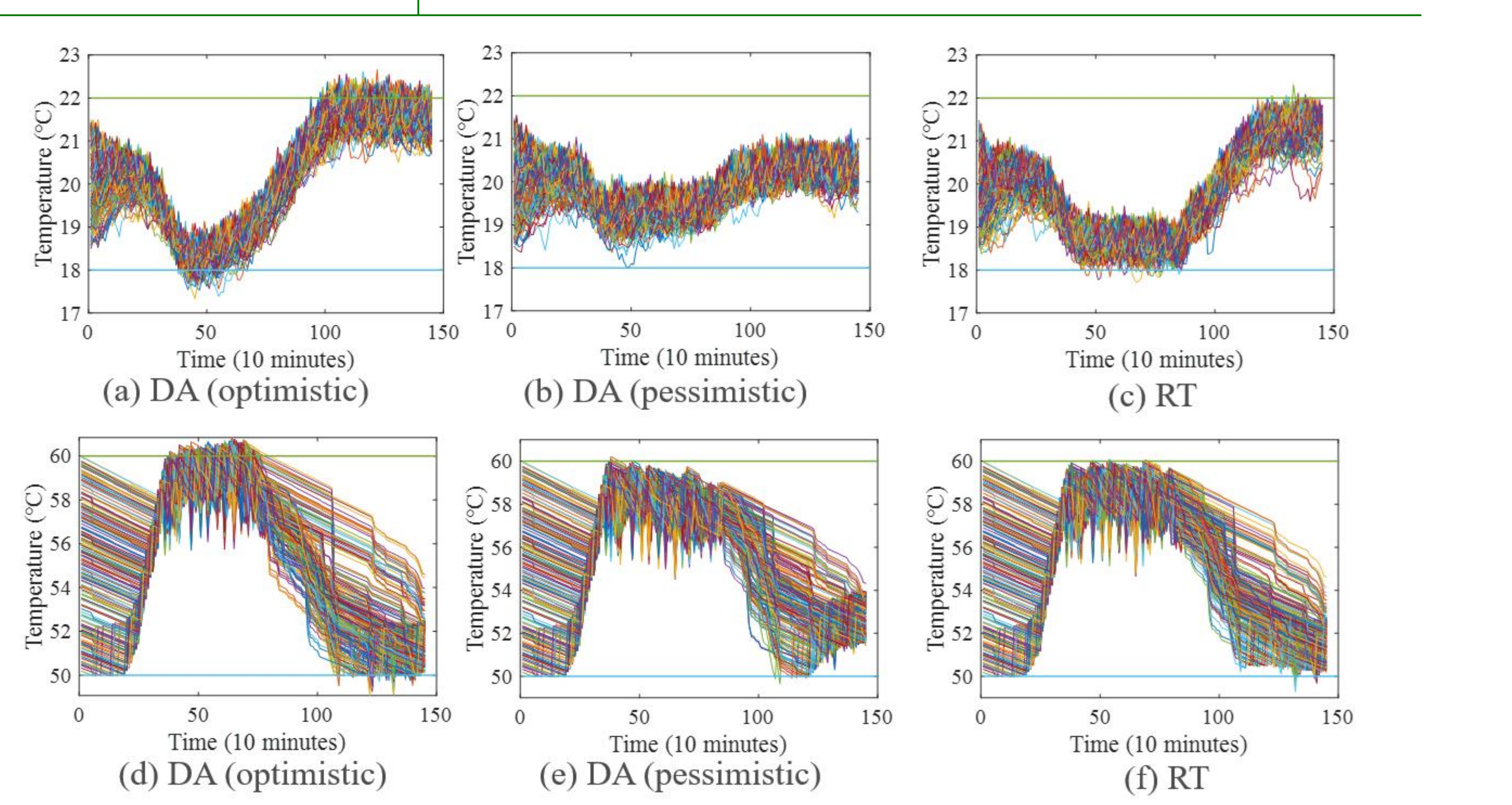
$$S_C = S_C - 1,\ p(n) = p(n) + p(m)$$

3.3 If $S_C > S_0$ (S_0 is the preferred scenario number), go back to **Step 3.1**; otherwise, terminate the algorithm.

Simulation Results



Profiles of total flexible load in day-ahead and real-time schedules.



Indoor temperature profiles of HVACs and water temperatures of EWHs.

Conclusions

- Uncertainties from PV power output, outdoor temperature, and individual consumption behaviors are modeled.
- The proposed hybrid interval-stochastic programming can effectively handle the uncertain bilevel problem.
- RHO scheme can mitigate the radicalness and conservativeness of the day-ahead schedule.