Time-variant Nonlinear Participation Factors Considering Resonances in Power Systems

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Background:

- The *participation factor* (**PF**) is a useful index to evaluate the contribution of each generator (or a state variable) to an oscillation mode.
- A *nonlinear participation factor* (**NPF**) is defined to evaluate the participation of a state variable into modal dynamics following a large disturbance, that gives considerations to nonlinearities up to a desired order.

$$\dot{\mathbf{x}} = f(\mathbf{x},\mathbf{u})$$

Taylor expansion up to 2nd order

$$\mathbf{x} = \mathrm{A}\mathbf{x} + \frac{1}{2}\left[\mathbf{x}^{\mathrm{T}}\mathrm{H}^{1}\mathbf{x} \quad \dots \quad \mathbf{x}^{\mathrm{T}}\mathrm{H}^{n}\mathbf{x}\right]^{\mathrm{T}}$$

Linearization

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \qquad \begin{cases} \mathbf{A}\boldsymbol{\phi}_i = \lambda_i \boldsymbol{\phi}_i \\ \boldsymbol{\psi}_i \mathbf{A} = \lambda_i \boldsymbol{\psi}_i \end{cases}$$

Product

PF:

$$p_{ki} \triangleq \phi_{ki}\psi_{ik}$$

Normal form method

NPF:

$$p_{2ki} = \phi_{ki}(\psi_{ik} + \psi_{2ikk})$$

$$p_{2kpq} = \phi_{2kpq}(\psi_{pk} + \psi_{2pkk})(\psi_{qk} + \psi_{2qkk})$$

$$\psi_{2mkk} = -\sum_{p=1}^{n}\sum_{q=p}^{n} h2_{pq}^{m}\psi_{pk}\psi_{qk} \qquad \phi_{2kpq} = \sum_{i=1}^{n} h2_{pq}^{i}\phi_{ki}$$

Problem Description:

- The scaling factor *α* can be canceled after the normalization of PFs but not for the NPFs :

$$p_{ki} = \alpha\phi_{ki}\psi_{ik} \qquad p_{2ki} = \alpha\phi_{ki}\psi_{ik} - \alpha^2\sum_{p=1}^{n}\sum_{q=p}^{n} h2_{pq}^{m}\psi_{pk}\psi_{qk}$$

- Discontinuity Between Linear and Nonlinear PFs:

$$\lim_{t\to\infty} p_{2ki} \neq p_{ki}$$

- The participation of the state in the combination is usually ignored in existing literature:

$$p_{2kpq} = \phi_{2kpq}(\psi_{pk} + \psi_{2pkk})(\psi_{qk} + \psi_{2qkk}) \approx 0$$

Table 1. Comparison of different PFs

Types	Time Performance	System Model	Mode
PF	Constant	Linear	Linear
NPF	Constant	Nonlinear	Linear
TNPF	Time-variant	Nonlinear	Nonlinear

Proposed Time-variant NPF:

- Replace scaling factor by a time decaying factor.
- Propose a nonlinear mode considering the influence from combination modes.
- Define the Time-variant NPF (TNPF) as:

$$p_2(t, f_{taregt}) = \int_0^{\infty} N(f_{taregt}, \sigma^2)\, p_g(f)\, df$$

$$p_g(f) = \begin{cases} p_{2ki}(t) & f \in \{\mathrm{Im}(\lambda_i)\} \\ p_{2kpq}(t) & f \in \{\mathrm{Im}(\lambda_p + \lambda_q)\} \\ 0 & \text{others} \end{cases}$$

where

$$p_{2ki}(t) = (\alpha_k e^{\lambda_i t})\phi_{ki}\psi_{ik} - \phi_{ki}\sum_{p=1}^{n}\sum_{q=p}^{n}(\alpha_p\alpha_q e^{(\lambda_p+\lambda_q)t})h2_{pq}^{m}\psi_{pk}\psi_{qk}$$

$$p_{2kpq}(t) = (\alpha_p\alpha_q e^{\lambda_p t + \lambda_q t})\phi_{2kpq}(\psi_{pk} + \psi_{2pkk})(\psi_{qk} + \psi_{2qkk})$$

Time decaying factor: $\alpha_k e^{\lambda_i t} \quad \alpha_p\alpha_q e^{(\lambda_p+\lambda_q)t}$

Nonlinear mode: $p_2(0, f_{taregt})$

Key Results and Conclusions:

- Tested on a two-area system.
- By introducing the convolution method, the nonlinear modes can be defined to calculate the TNPF (Fig. 1).
- The time decaying factor in TNPF allows a smooth transition from the NPF to the linear PF for an oscillating power system subject to a large disturbance and also addresses resonances (Fig. 2).
- **Conclusion:** TNPF can address the existing problems with PF and NPF.

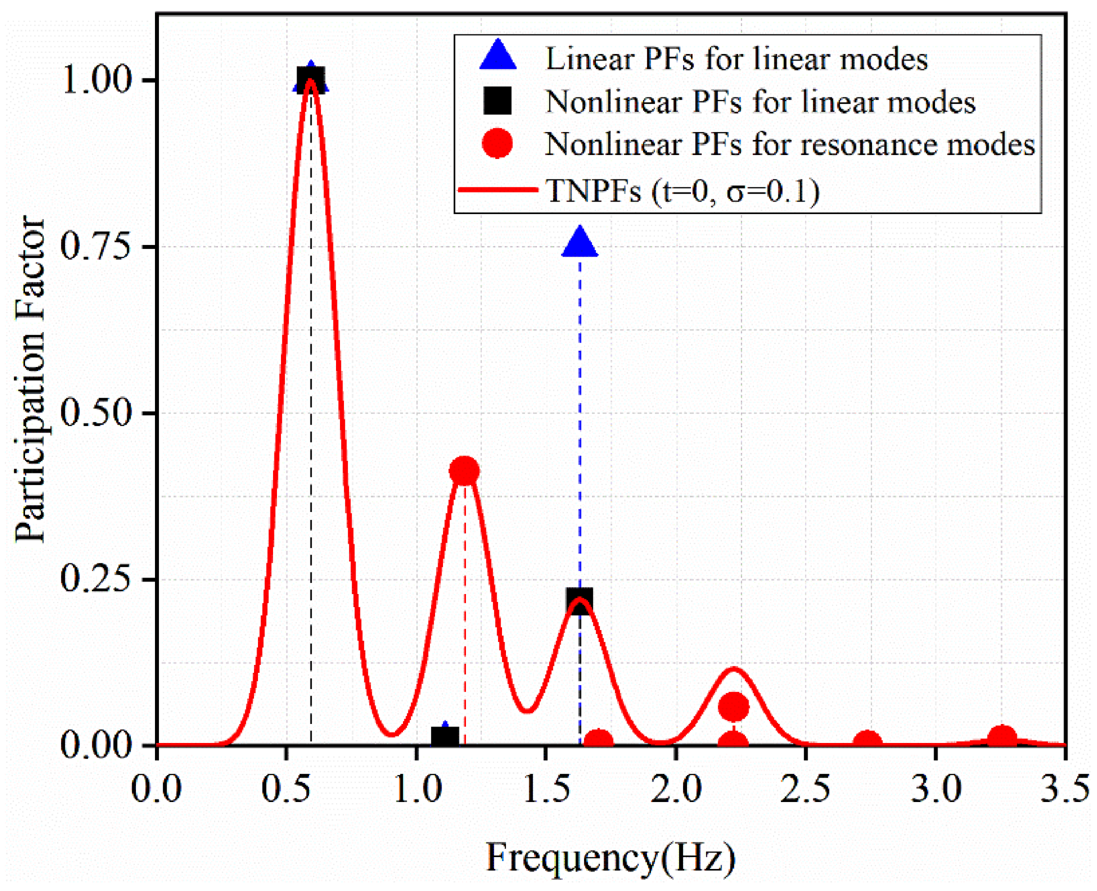

Fig. 1. Spectrum of generator 1

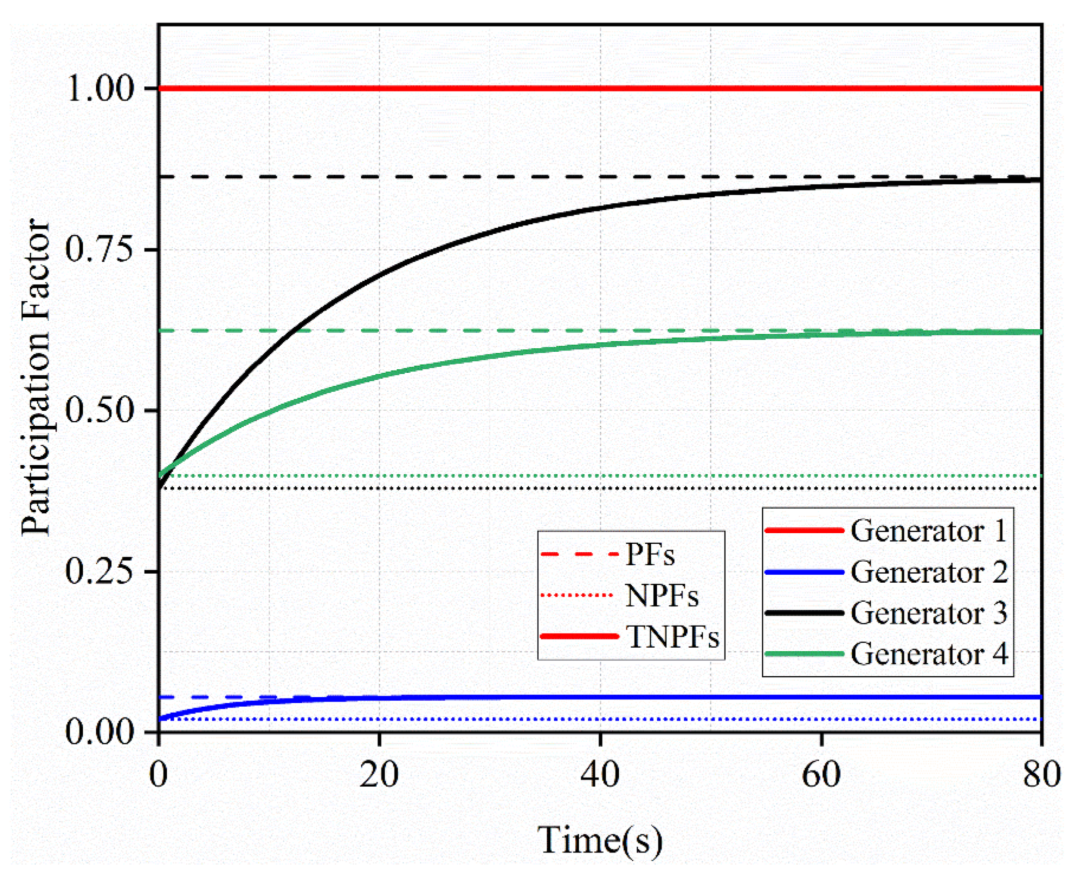

Fig. 2. The trajectories of PF, NPF and TNPF for 0.59 Hz mode

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