Semi-Analytical Electromagnetic Transient Simulation Using Differential Transformation

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Introduction

	Transient stability simulation	Electromagnetic transient simulation
Time step	Milliseconds	Microseconds
Model	Positive-sequence, three-phase balanced model	Detailed three phase model
Voltages and currents	Phasor value	Instantaneous value
Dynamics of interest	Slower dynamics, e.g., electromechanical interactions among generators	Fast dynamics, e.g., electromagnetic interactions among power electronics devices
Frequency	Power frequency	Wide range of frequency
Advantages	Fast and efficient	Capture detailed fast dynamics, simulate unbalanced faults
Disadvantages	Suitable only for low frequency dynamics	High computation cost

[1] J. Mahseredjian, V. Dinavahi and J. A. Martinez, "Simulation Tools for Electromagnetic Transients in Power Systems: Overview and Challenges," *IEEE Trans. Power Del.,* vol. 24, no.3, pp. 1657-1669, July 2009.

State-space EMT simulation

State-space method

x is the state variables vector,
u is the system input vector,
y is the system output vector.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \qquad (1)$$

Example

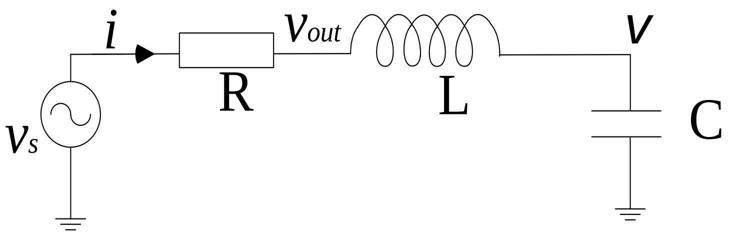

Fig.1. An RLC circuit with an ideal voltage source

$$\begin{bmatrix} \dot{i} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_s \qquad (2)$$

$$v_{out} = -Ri + v_s$$

Work in this paper:
Based on **state-space-represented** EMT simulations, this paper proposes a **semi-analytical-solution** method that repeatedly utilizes a high-order approximate solution of the EMT equations at **longer time steps.**

[2] N. R. Watson and J. Arrillaga, "Power systems electromagnetic transient simulation," Inst. Eng. Technol. Power and Energy, ser. 39, 2007.

Differential transformation method

Considering a continuous function *f*(t), the k^{th} order differential transform (DT) of *f*(t) is defined by

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t0} \quad (3)$$

Also, from the Taylor series

$$f(t) = \sum_{k=0}^{\infty} F(k)\,(t-t_0)^k \quad (4)$$

Replace all the variables in equations by their Taylor series composed of DT and equal the like terms of $(t–t_0)^k$ in the equation, then linear equations of different order DTs can be established.

Original continuous functions $f(t)$ $g(t)$ $h(t)$ → k^{th} order DT $F(k)$ $G(k)$ $H(k)$

Original continuous functions	k^{th} order DT
$f(t) = c$	$F(k) = c\eta(k),\ \eta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$
$f(t) = cg(t)$	$F(k) = cG(k)$
$f(t) = g(t) \pm h(t)$	$F(k) = G(k) \pm H(k)$
$f(t) = g(t)h(t)$	$F(k) = \sum_{m=0}^{k} G(k)H(k-m)$
$f(t) = \dfrac{dg(t)}{dt}$	$F(k) = (k+1)G(k+1)$
$f(t) = \sin(h(t))$	$F(k) = \sum_{m=0}^{k-1} \frac{k-m}{k} G(m)H(k-m)$
$g(t) = \cos(h(t))$	$G(k) = -\sum_{m=0}^{k-1} \frac{k-m}{k} F(m)H(k-m)$

Finally, any state variables in the equation can be approximated by their DTs up to the k^{th} order, e.g.

$$f(t) \approx \sum_{k=0}^{i} F(k)\,(t-t_0)^k \quad (5)$$

[3] Y. Liu and K. Sun, "Solving power system differential algebraic equations using differential transformation," *IEEE Trans. Power Syst.*, vol. 35, no. 3, pp. 2289-2299, May. 2020.

Case study

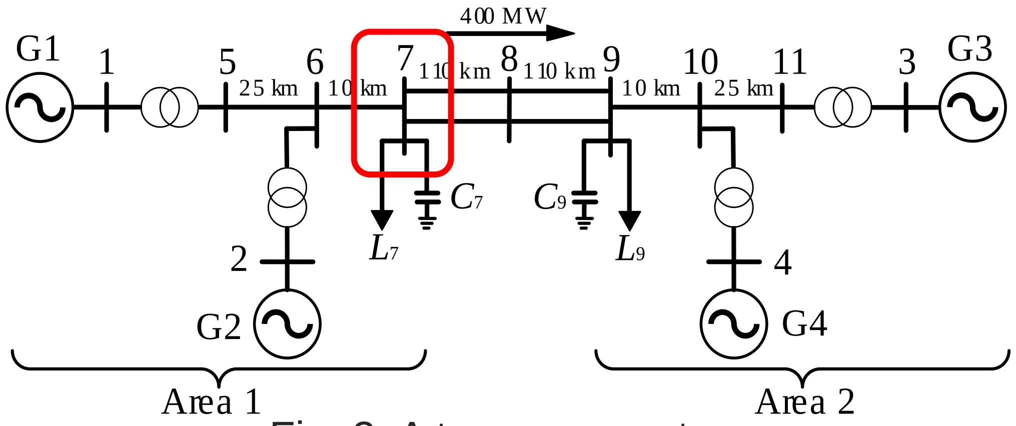

Fig. 2. A two-area system

Steady state : 0-1 s

During fault : Bus 7 grounded for 5 cycles

Post fault : Fault cleared

Post fault simulation lasting for 2 seconds

20th-order DT method using a time step of 100 μs is compared with the benchmark

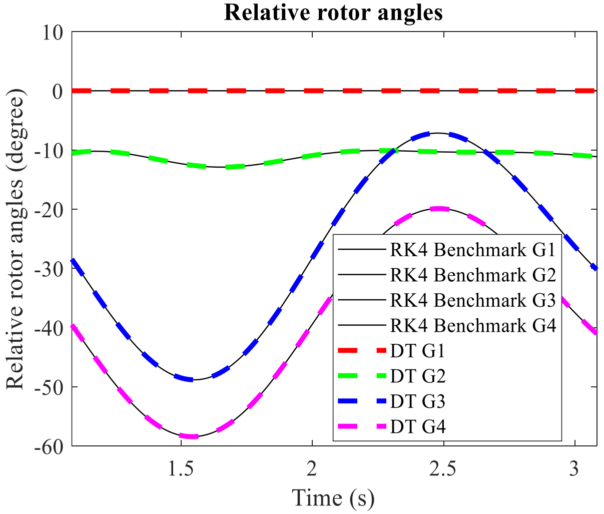

Fig. 3. Synchronous generator relative rotor angle

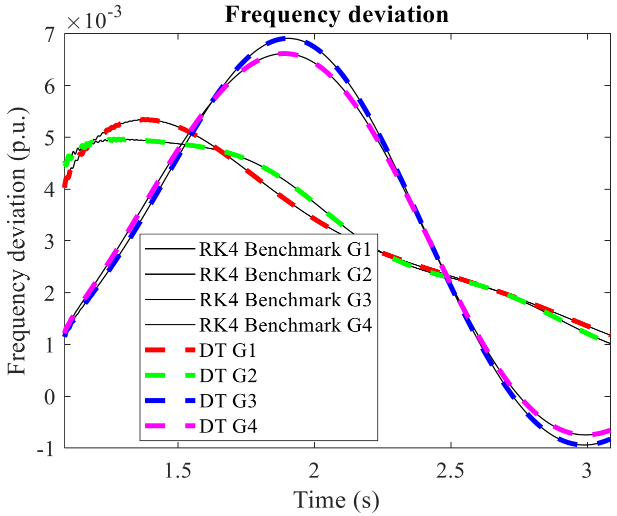

Fig. 4. Synchronous generator frequency deviation

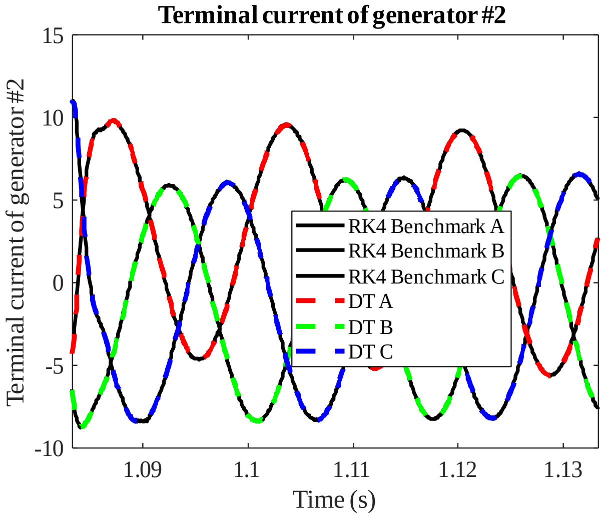

Fig. 5. Three-phase terminal current of generator #2

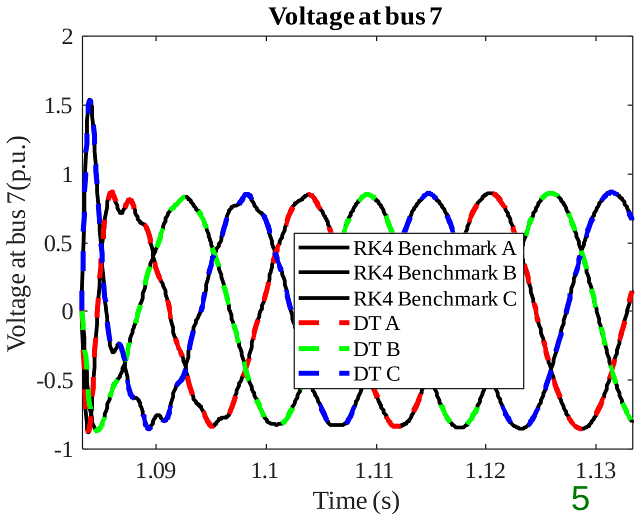

Fig. 6. Three-phase voltage at bus #7

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Case study

(1) Comparison between the differential transformation with different orders

Error (p.u.)	DT (10th order)		DT (20th order)		DT (30th order)		DT (40th order)	
	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)
10^{-2}	72	81	200	58	330	53	412	58
10^{-3}	58	100	180	64	306	57	382	62
10^{-4}	46	1162	158	73	284	62	365	68

(1) High order DT could enlarge the time step.
(2) When the order is too high, the burden of computing high order terms decreases the overall efficiency.

(2) Comparison between the 30th order differential transformation with conventional numerical methods

Error (p.u.)	Modified Euler		4th oder Runge kutta		DT (30th order)		Trapezoidal-rule	
	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)	Time step (μs)	Time cost (s)
10^{-2}	1.0	1045	10	200	330	53	400	92
10^{-3}	0.5	2100	5	360	306	57	400	230
10^{-4}	0.5	4215	3	550	284	62	400	600

(1) The DT based approach enables large time steps and thus reduces time costs.
(2) Trapezoidal-rule method is A-stable and enables large time step, but iterations are required to achieve high accuracy, and increases time cost.

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