

# Incentive Compatible Power Market Design by Indirect Groves Mechanisms

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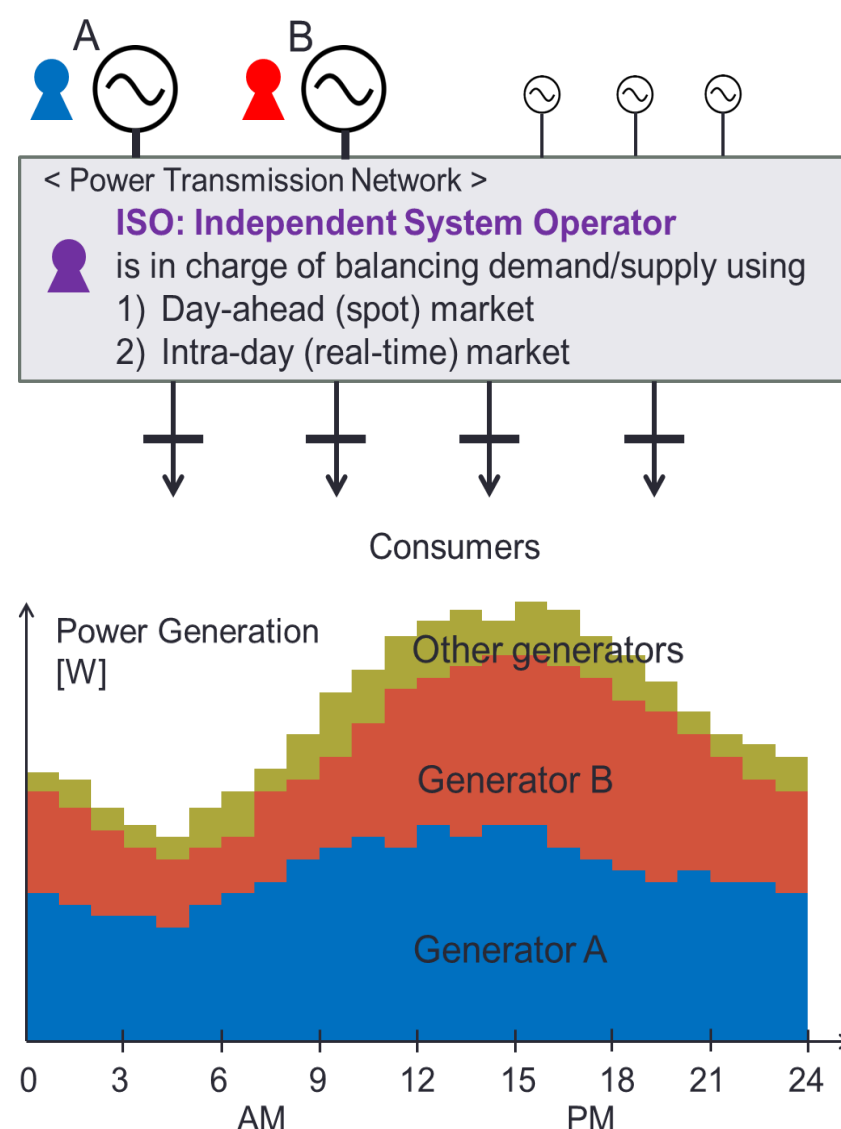
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## Abstract

In an inappropriately designed oligopolistic power supply market, stakeholders are incentivized to exercise their market power to manipulate the market in order to maximize their own profit, resulting in a significant loss of social welfare. We discuss this issue in the framework of the indirect mechanism design theory, aiming at implementing socially optimal actions by power generators in a game theoretic equilibrium. We show that indirect Groves mechanisms are not only sufficient but also necessary to implement efficient distributed algorithms in ex-post Nash equilibrium, which can be viewed as a generalization of the Green-Laffont theorem. In particular, we demonstrate that the classical tâtonnement process to find the socially optimal solution can be made incentive compatible by introducing a reward function from the (indirect) Groves class.

## Background

Power supply market is often oligopolistic.



**Question:** Is it possible to design a day-ahead market mechanism in which generators' selfish bidding strategies in an effort to maximize their own profit lead to a socially optimal outcome?

## Naïve Market Mechanism

### ("Clearing-price" mechanism)

**Step 1:** Each generator participates tâtonnement process to determine generation share.

**Step 2:** Compute rewards by

$$\pi_i = \sum_{t=1}^T p^*(t) x_i^*(t). \quad (\text{No price discrimination})$$

**Definition:** A market mechanism is said to be **ex-post Nash incentive compatible** if participating tâtonnement process as designated is a Nash equilibrium for the power generators.

**Remark:** Although the "clearing-price" mechanism seems natural, it is not incentive compatible in general – a strategic generator with market power can be better-off by manipulating the market clearing price, resulting in a significant loss of social welfare.

## Proposed Market Mechanism

### (Indirect Groves mechanism)

**Step 1:** Each generator participates tâtonnement process to determine generation share.

**Step 2:** Compute rewards by

$$\pi_i = \sum_{j \neq i} \sum_{t=1}^T l_j(x_j^*(t), u_j^*(t)) + k_i$$

where  $k_i$  is a quantity that is not dependent on the  $i$ -th generator's strategy.

**Theorem 1:** (Sufficiency) Indirect Groves mechanism is ex-post Nash incentive compatible.

**Theorem 2:** (Necessity [1]) Under mild assumptions, an efficient (=maximizing social welfare) mechanism is ex-post Nash incentive compatible only if it is in the class of indirect Groves mechanisms.

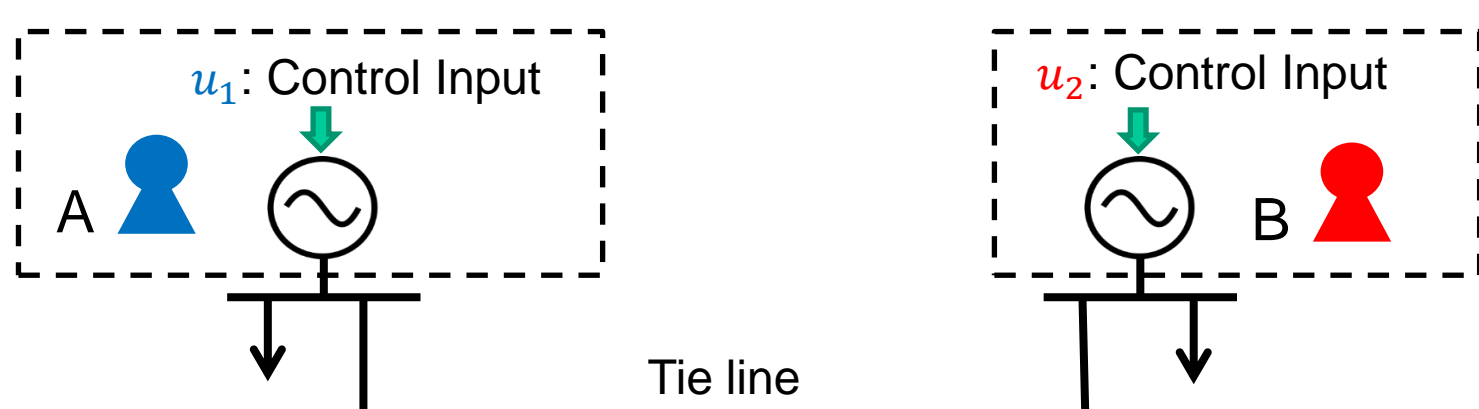
**Remark 1:** Direct vs. Indirect mechanisms (e.g., [3])

- Direct mechanisms induce "truth-telling" by agents.
- Indirect mechanisms induce "faithful actions" by agents.

**Remark 2:** Theorem 2 can be viewed as a generalization of the Green-Laffont theorem to indirect mechanisms.

## Problem Set-up

**Example:** Power supply market with two generators.



$$\begin{aligned} x_1(t+1) &= A_1 x_1(t) + B_1 u_1(t) & x_2(t+1) &= A_2 x_2(t) + B_2 u_2(t) \\ \delta_1(t+1) &= \delta_1(t) + f_1(t) & \delta_2(t+1) &= \delta_2(t) + f_2(t) \\ H_1 f_1(t+1) &= H_1 f_1(t) + C_1 x_1(t) & H_2 f_2(t+1) &= H_2 f_2(t) + C_2 x_2(t) \\ &+ (\delta_2(t) - \delta_1(t))/X & &+ (\delta_1(t) - \delta_2(t))/X \end{aligned}$$

$x_i$  : Deviation in power generation by the  $i$ -th generator.

$\delta_i$  : Deviation in mechanical angle of the  $i$ -th generator.

$f_i$  : Frequency deviation at the  $i$ -th generator.

ISO aims at maximizing social welfare

$$L_0 = \sum_{t=1}^T \left[ \underbrace{h(f_1(t), f_2(t), \delta_1(t), \delta_2(t))}_{\text{Quality of service}} + \sum_{i=1,2} \underbrace{l_i(x_i(t), u_i(t))}_{\text{Utility (negated cost) functions}} \right]$$

Assume that utility functions  $l_i$  are not known to ISO, and that the optimal generation share  $x_i^*(t)$  must be computed by the following tâtonnement process.

### (Tâtonnement process)

#### Repeat

- Each generator updates generation share  $x_i$  by  $\{x_i(t), u_i(t)\}_{t=1}^T = \arg\max \left( \sum_{t=1}^T l_i(x_i(t), u_i(t)) + p(t)x_i(t) \right)$
- Price update by  $p(t) \leftarrow p(t) + \eta(\text{Demand}(t) - \sum_i x_i(t))$

#### Until converge



If all generators participate tâtonnement process faithfully, the process converges to a socially optimal share  $x^*$  and the market clearing price  $p^*$ .

**Generator A** maximizes net profit

$$L_1 = \sum_{t=1}^T l_1(x_1(t), u_1(t)) + \pi_1$$

**Generator B** minimizes net profit

$$L_2 = \sum_{t=1}^T l_2(x_2(t), u_2(t)) + \pi_2$$

**Task:** Design reward functions  $\pi_i$  so that no strategic generator is incentivized to deviate from implementing tâtonnement process faithfully.

## Pros & Cons

Indirect Groves mechanisms

#### Pros

- Incentive compatibility
- Distributed computation
- No need to report utility functions

#### Cons

- Price discrimination
- Communication complexity
- Budget balance

## Conclusion

### Summary

- "Clearing-price" mechanism fails to be incentive compatible.
- (Indirect) Groves mechanism faithfully implements tâtonnement process.
- (Indirect) Groves mechanism is the only mechanism that implements efficient decision rules in ex-post Nash equilibrium.

### Future work

- Budget balance and individual rationality
- Extension to real-time market (e.g., MPC with receding planning horizon)
- Strategic collusions

## Reference

[1] Tanaka, Farokhi, and Langbort "Faithful Implementations of Distributed Algorithms and Control Laws" arXiv:1309.4372v3, 2014

[2] Tanaka, Farokhi, and Langbort "A Faithful Distributed Implementation of Dual Decomposition and Average Consensus Algorithms" CDC 2013

[3] Parkes and Shneidman "Distributed implementations of Vickrey-Clarke-Groves mechanisms" Conf. on Autonomous Agents and Multiagent Systems 2004