Distributed Algorithms for Wide-Area Monitoring of Power Systems

Theory, Experiments, and Open Problems

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2015 JST-NSF-DFG-RCN Workshop

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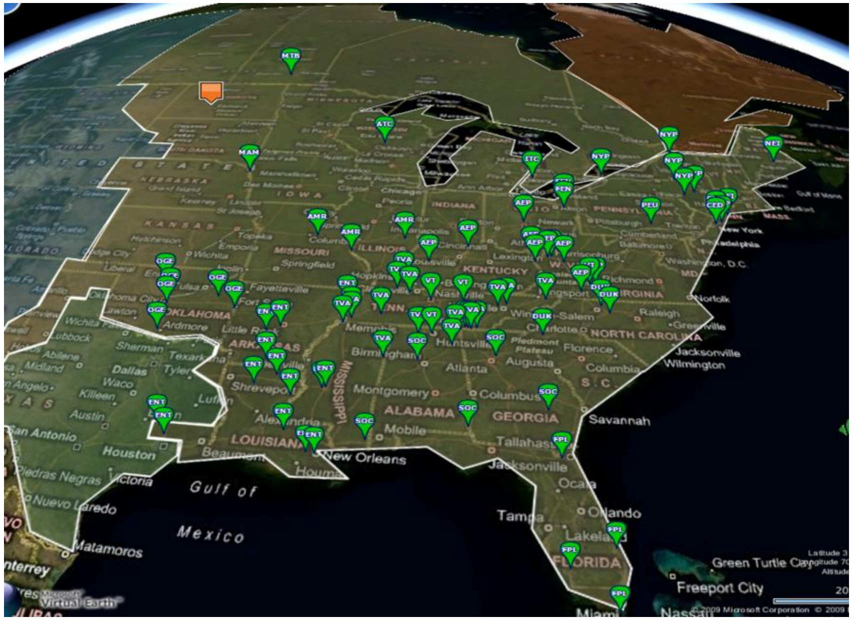
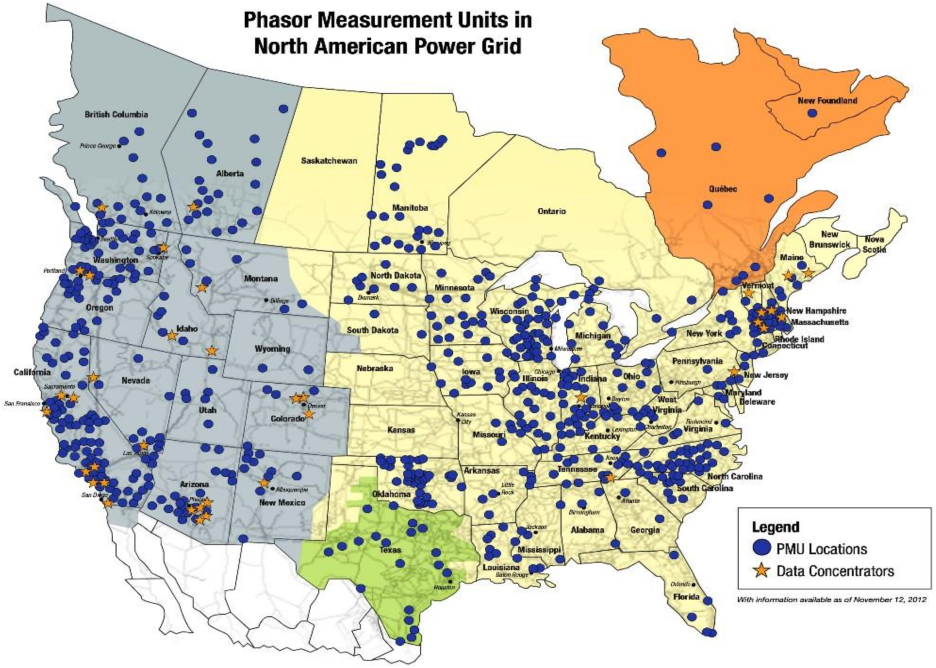
Increasing Volumes of PMU Data



2008: Only 40 PMUs in the entire east coast

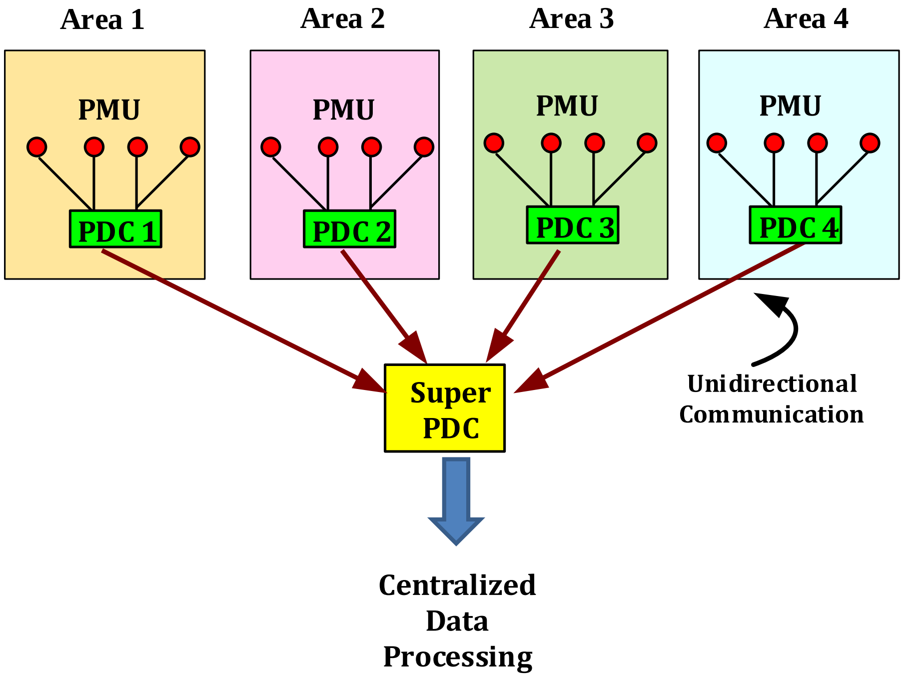


2015: More than 1200 PMUs across USA
(Nearly 52 PMUs only in North Carolina)

- Massive volumes of PMU data need to be transported from one part of the grid to another for monitoring and control
- Needs a highly reliable and resilient communication infrastructure
- Centralized processing will not be tenable
- Need combination of distributed monitoring spread over the entire system

Centralized vs Distributed Architectures

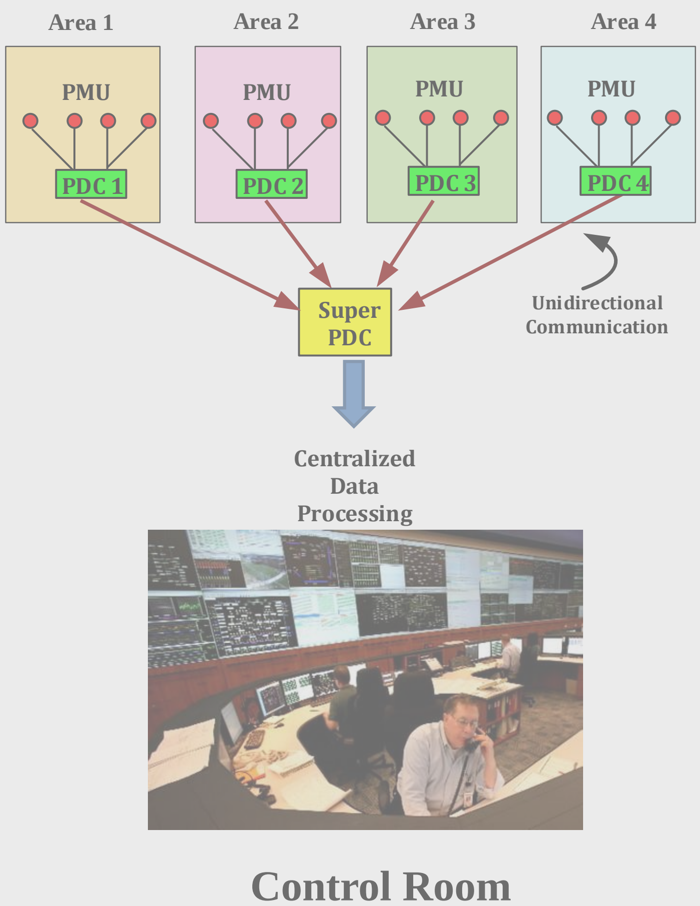
Centralized WAMS



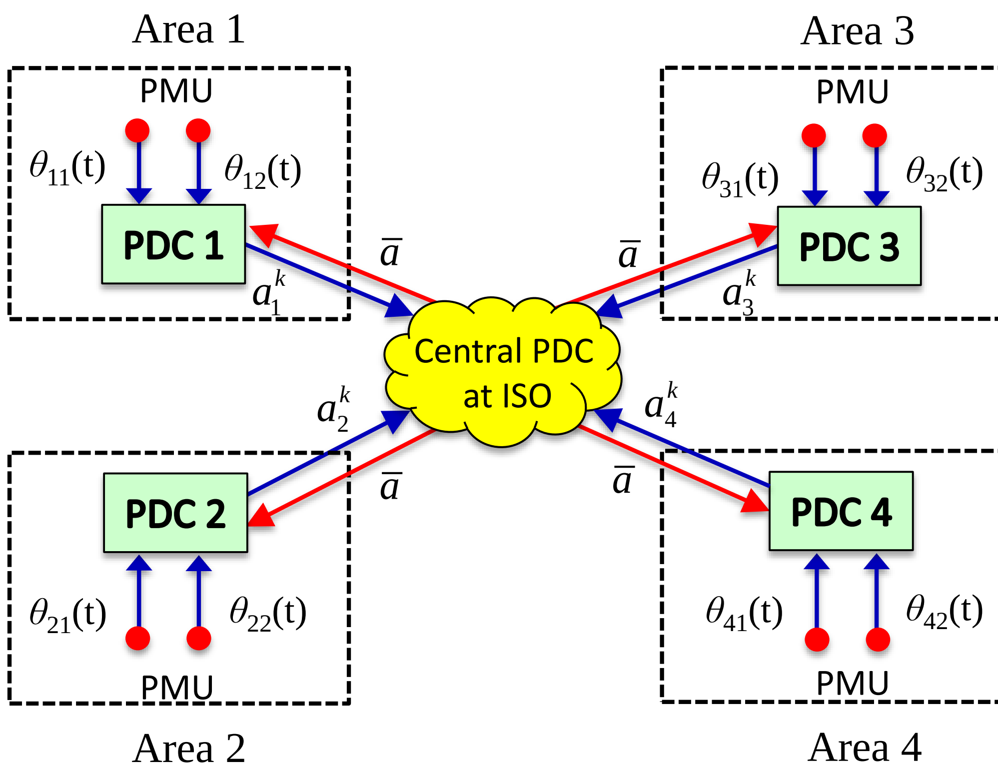
Control Room

Centralized vs Distributed Algorithms

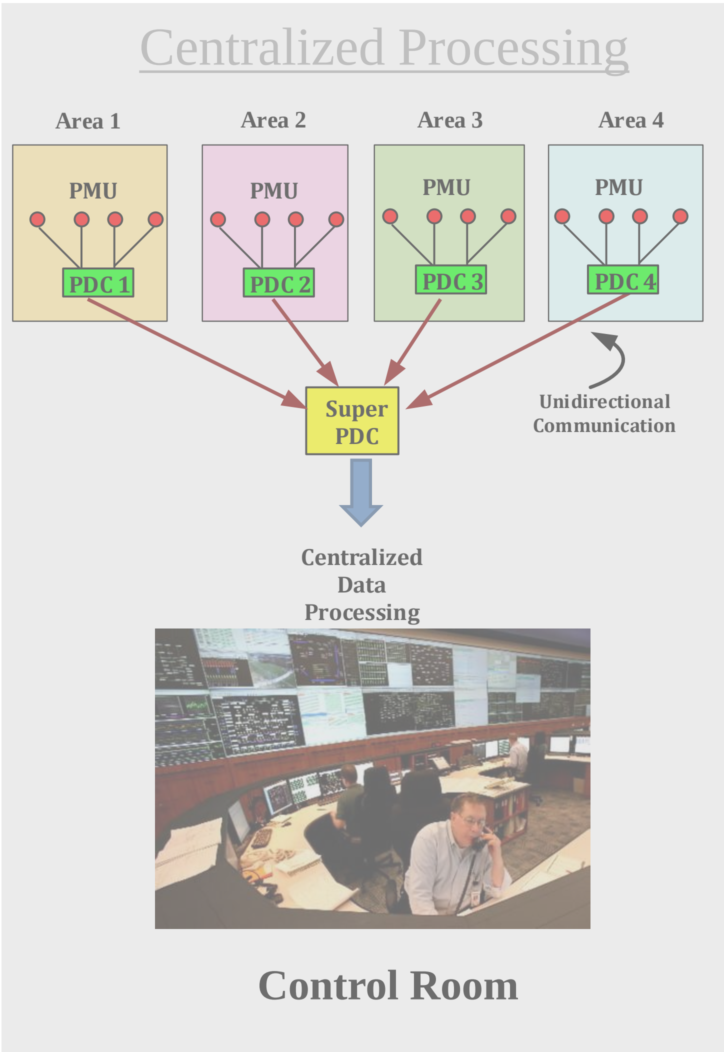
Centralized WAMS

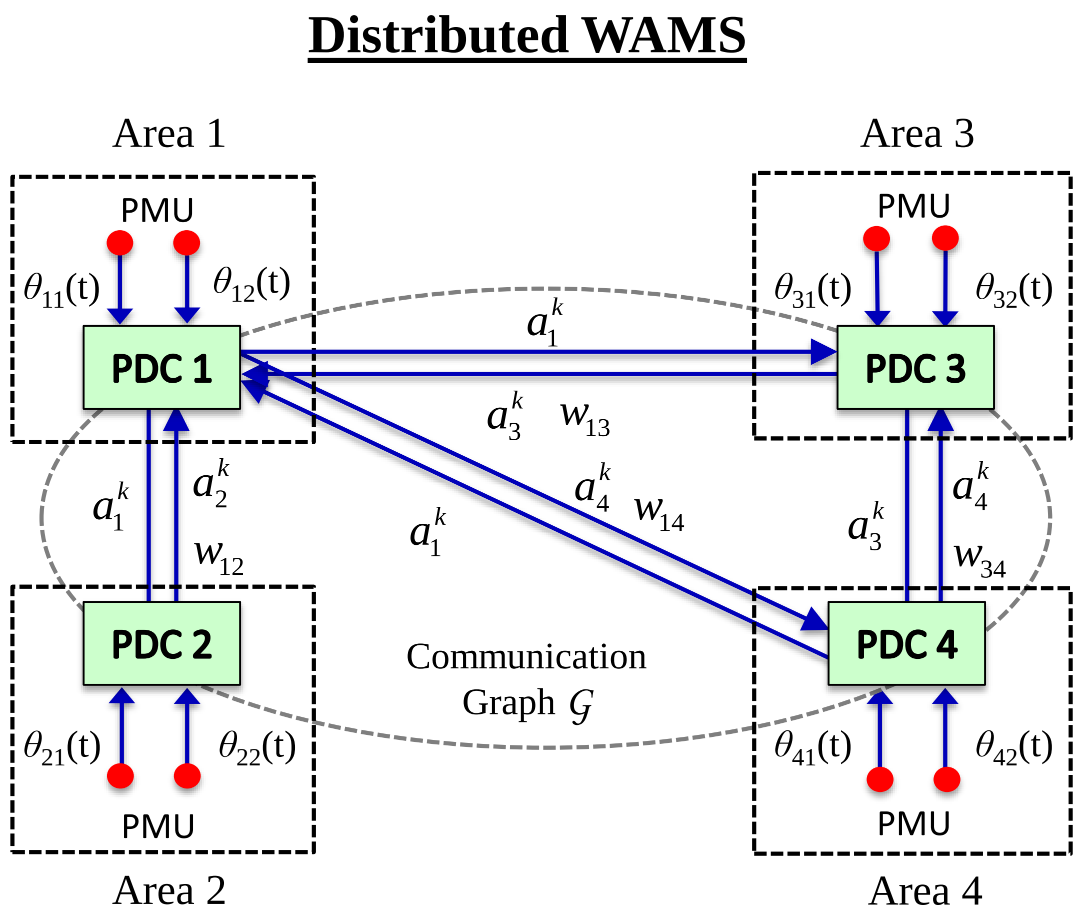


Semi-Distributed WAMS

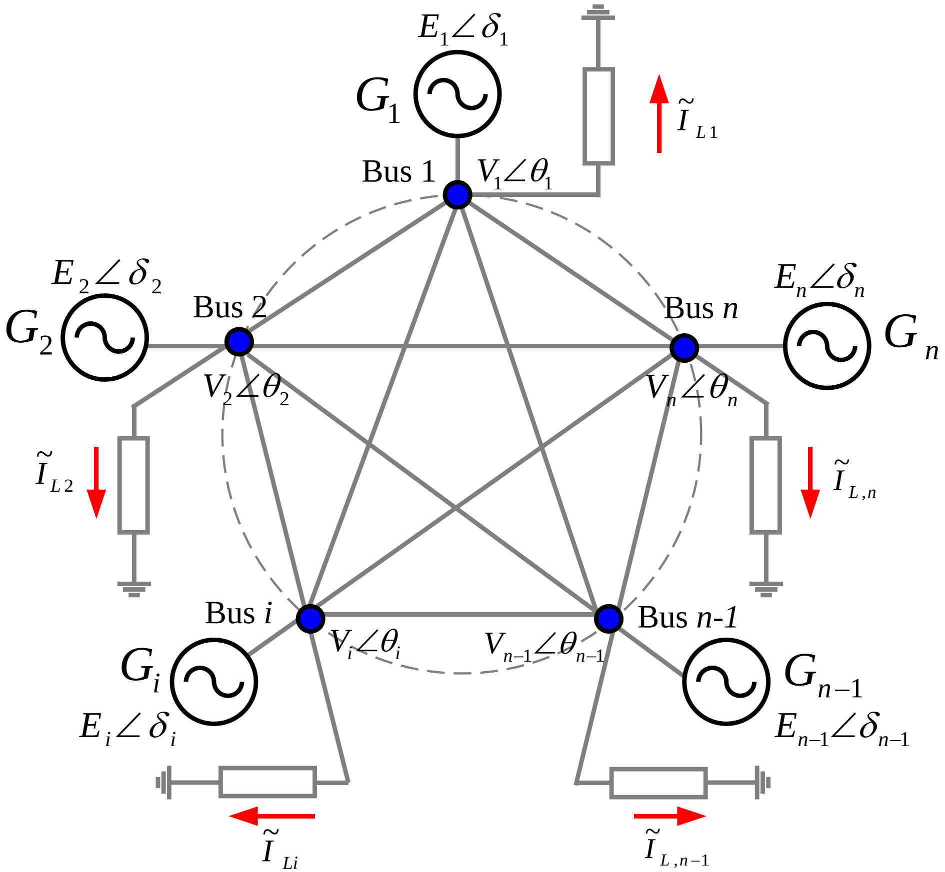


Centralized vs Distributed Algorithms





Wide-Area Oscillation Estimation



Output Equation

$$y = \mathrm{col}_{i\in S}(\Delta V_i,\, \Delta\theta_i).$$

Swing equation model:

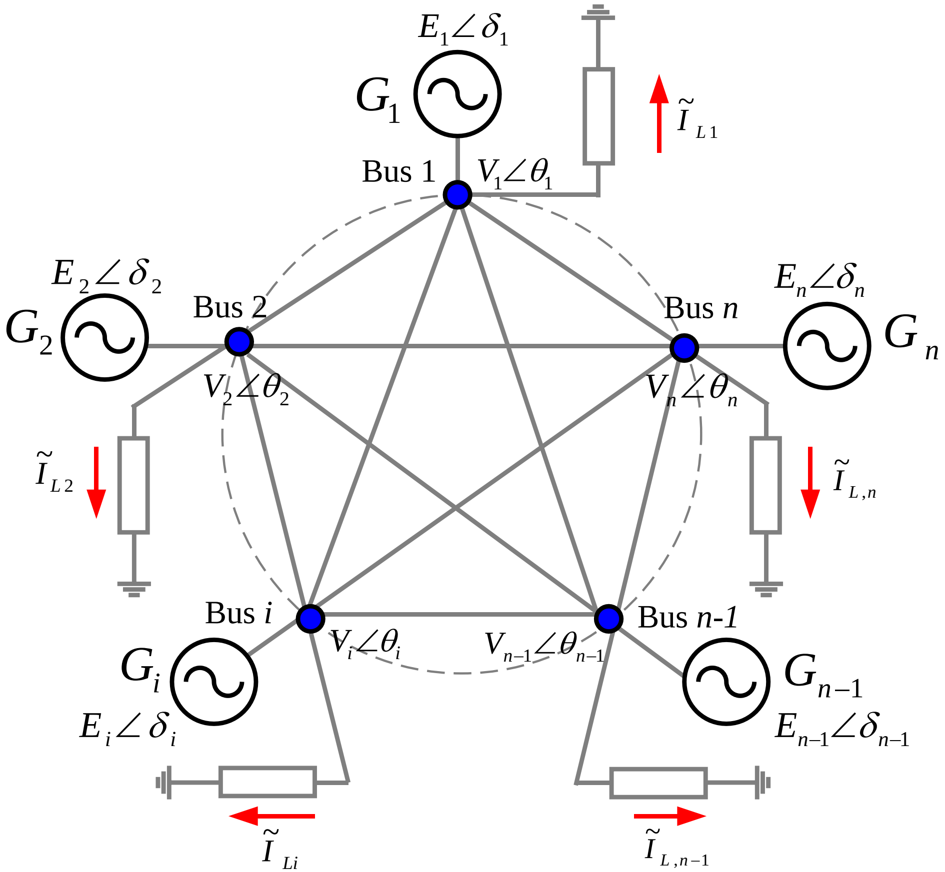
$$\begin{bmatrix} \Delta\dot{\delta} \\ M\Delta\dot{\omega} \\ \Delta\dot{E} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -L(G) & -D & -P \\ K & 0 & J \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \mathrm{col}_{i=1(1)n}(\gamma_i) \\ \mathrm{col}_{i=1(1)n}(\rho_i) \end{bmatrix}}_{\text{due to load}} + \begin{bmatrix} 0 & 0 \\ 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} \Delta P_m \\ \Delta E_F \end{bmatrix}$$

$L(G)$ = fully connected network graph

Controllable inputs

A. Chakrabortty & P. Khargonekar, ACC 2013

Wide-Area Oscillation Estimation



Output Equation

$$y = \mathrm{col}_{i\in S}(\Delta V_i,\ \Delta\theta_i).$$

Wide-Area Oscillation Monitoring:

→ Estimate the eigenvalues and eigenvectors of $M^{-1}L$ using $y(t)$

Swing equation model:

$$\begin{bmatrix}\Delta\dot{\delta}\\ M\Delta\dot{\omega}\\ \Delta\dot{E}\end{bmatrix} = \begin{bmatrix}0 & I & 0\\ -L(G) & -D & -P\\ K & 0 & J\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta E\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \mathrm{col}_{i=1(1)n}(\gamma_i)\\ \mathrm{col}_{i=1(1)n}(\rho_i)\end{bmatrix}}_{\text{due to load}} + \begin{bmatrix}0 & 0\\ 0 & I\\ I & 0\end{bmatrix}\begin{bmatrix}\Delta P_m\\ \Delta E_F\end{bmatrix}$$

$L(G)$ = fully connected network graph

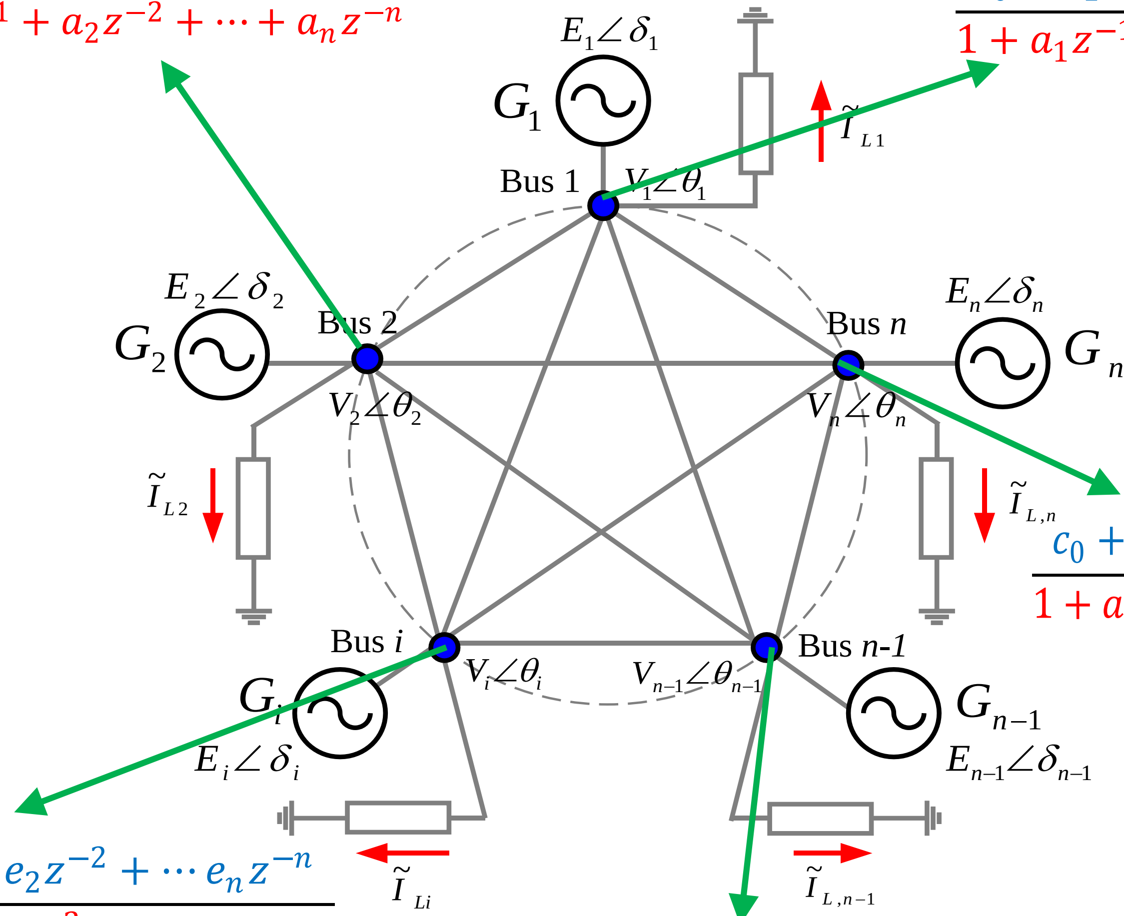
Controllable inputs

A. Chakrabortty & P. Khargonekar, ACC 2013

Wide-Area Oscillation Estimation

$$\frac{f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots f_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$

$$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$



$$\frac{c_0 + c_1 z^{-1} + c_2 z^{-2} + \cdots c_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$

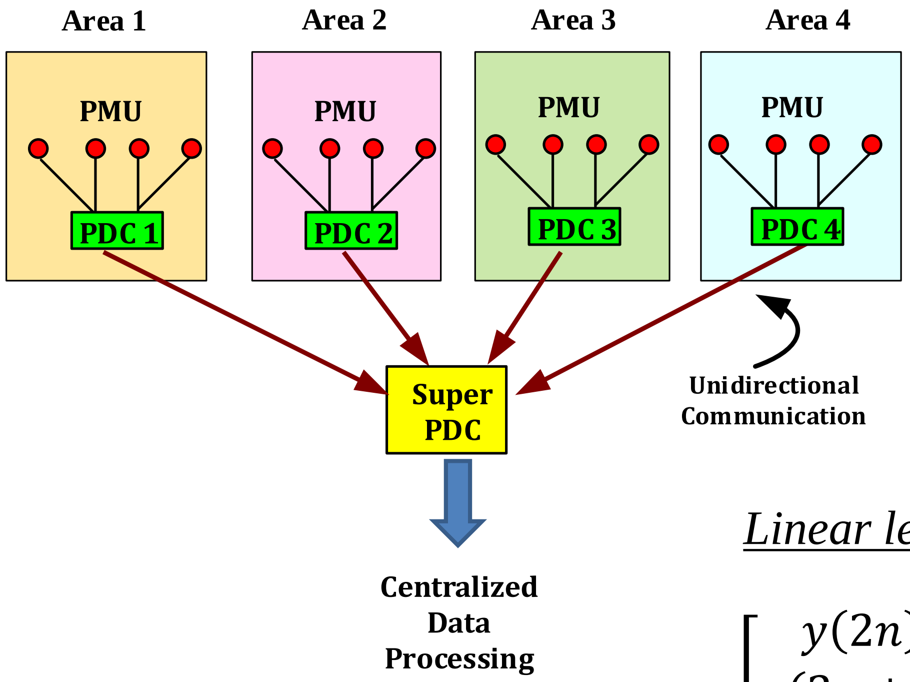
$$\frac{e_0 + e_1 z^{-1} + e_2 z^{-2} + \cdots e_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$

$$\frac{d_0 + d_1 z^{-1} + d_2 z^{-2} + \cdots d_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}$$

A. Chakrabortty & P. Khargonekar, ACC 2013

Wide-Area Oscillation Estimation

Centralized:



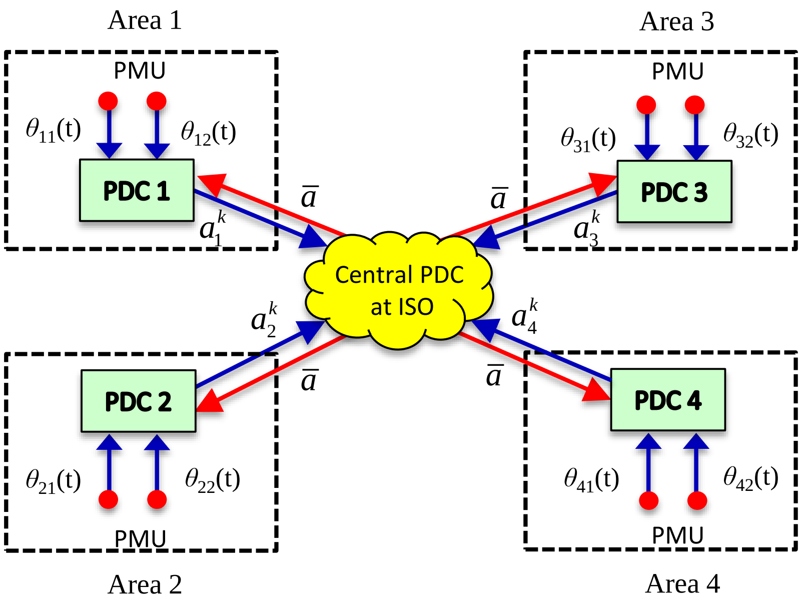
Linear least squares problem:

$$\underbrace{\begin{bmatrix} y(2n) \\ y(2n+1) \\ \vdots \\ y(2n+l) \end{bmatrix}}_{c} = \underbrace{\begin{bmatrix} y(2n-1) & \cdots & y(0) \\ \vdots & \ddots & \vdots \\ y(2n-1+l) & \cdots & y(l) \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_n \end{bmatrix}}_{a}$$

$$\longrightarrow \quad \hat{a} = \arg\min_{a} \|Ha - c\|^2$$

Wide-Area Oscillation Estimation

Distributed:



Multiple Computational Areas

$$\text{Area 1: } \hat{\theta}_1 = \{\theta_{30}, \theta_{66}\} \rightarrow (\hat{H}_1 = \begin{bmatrix} H_{30} \\ H_{66} \end{bmatrix}, \hat{\mathbf{c}}_1 = \begin{bmatrix} \mathbf{c}_{30} \\ \mathbf{c}_{66} \end{bmatrix})$$

$$\text{Area 2: } \hat{\theta}_2 = \{\theta_{16}, \theta_{53}\} \rightarrow (\hat{H}_1 = \begin{bmatrix} H_{16} \\ H_{53} \end{bmatrix}, \hat{\mathbf{c}}_1 = \begin{bmatrix} \mathbf{c}_{16} \\ \mathbf{c}_{53} \end{bmatrix})$$

$$\text{Area 3: } \hat{\theta}_3 = \{\theta_{68}\} \rightarrow (\hat{H}_3 = H_{68}, \hat{\mathbf{c}}_3 = \mathbf{c}_{68})$$

$$\text{Area 4: } \hat{\theta}_4 = \{\theta_{56}\} \rightarrow (\hat{H}_4 = H_{56}, \hat{\mathbf{c}}_4 = \mathbf{c}_{56})$$

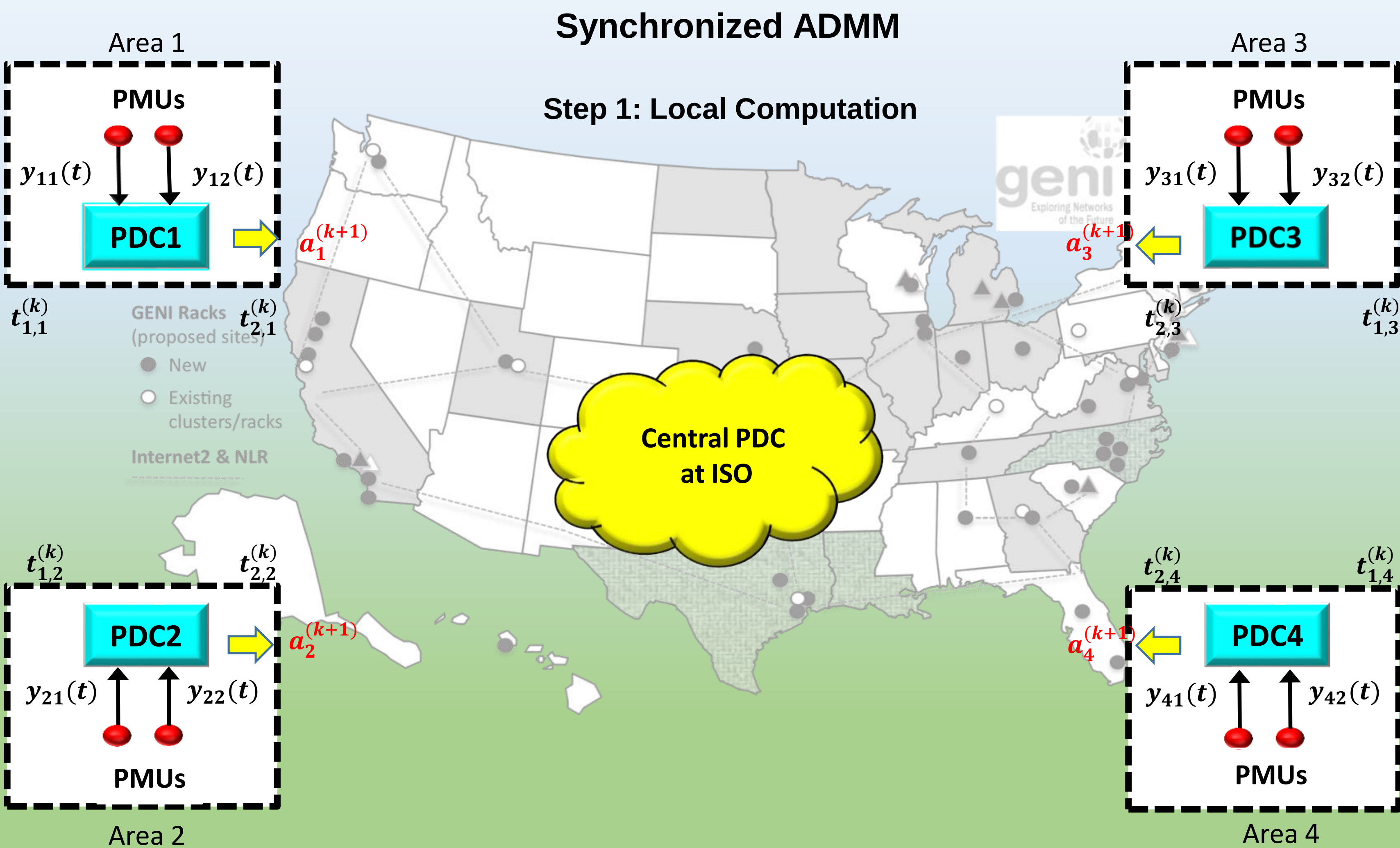
Global Consensus Problem:

$$\underset{\mathbf{a}_1,\ldots,\mathbf{a}_N,\mathbf{z}}{\text{minimize}} \sum_{i=1}^{N} \frac{1}{2} \left\| \hat{H}_i \mathbf{a}_i - \hat{\mathbf{c}}_i \right\|_2^2$$

$$\text{subject to } \mathbf{a}_i - \mathbf{z} = 0, \text{ for } i = 1, \ldots, N$$

Solve in a distributed way using:

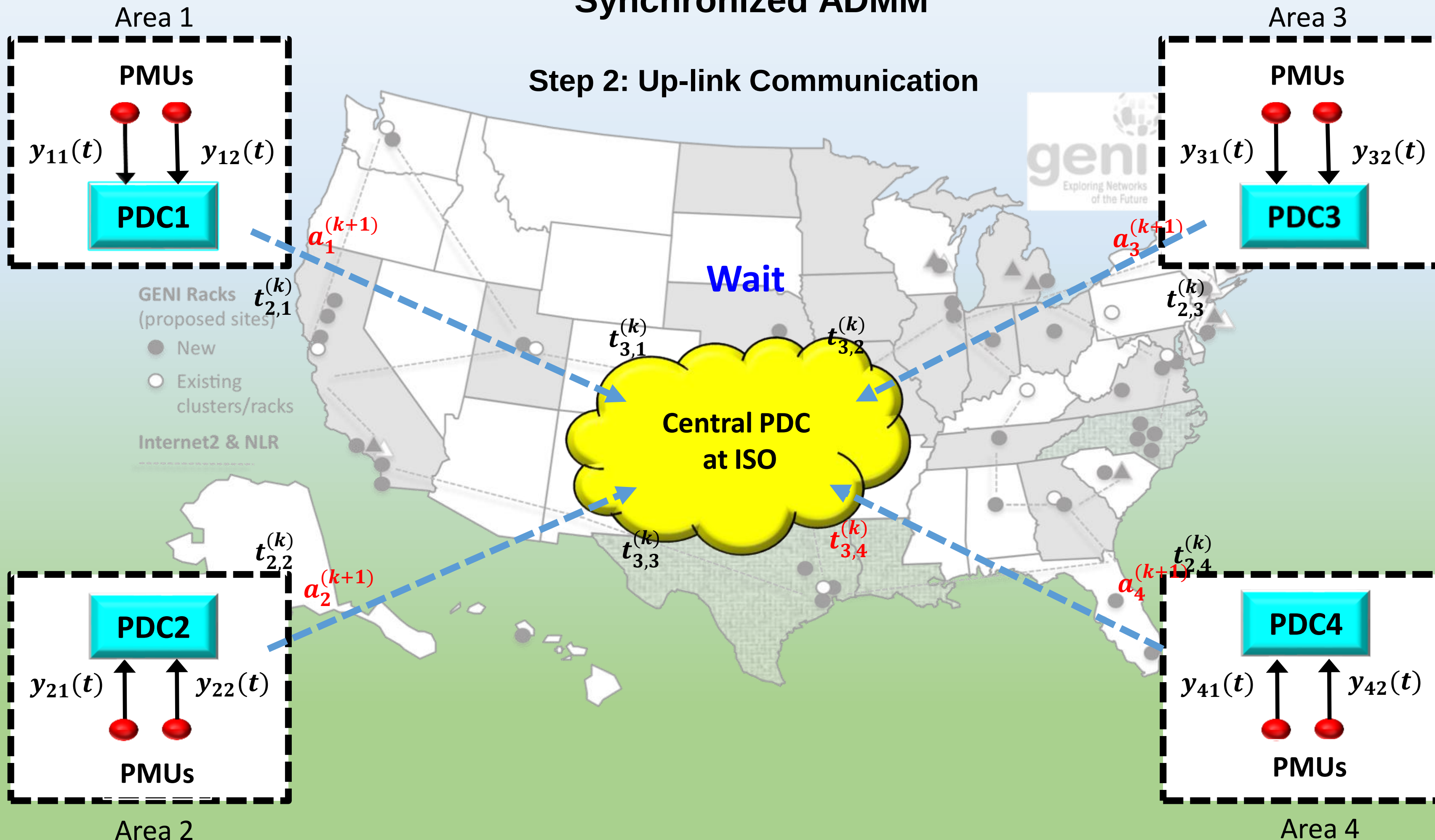
Alternating Direction Method of Multipliers (ADMM)



Synchronized ADMM
Step 1: Local Computation
Area 1
PMUs
$y_{11}(t)$
$y_{12}(t)$
PDC1
$a_1^{(k+1)}$
$t_{1,1}^{(k)}$
$t_{2,1}^{(k)}$
Area 3
PMUs
$y_{31}(t)$
$y_{32}(t)$
PDC3
$a_3^{(k+1)}$
$t_{2,3}^{(k)}$
$t_{1,3}^{(k)}$
GENI Racks
(proposed sites)
New
Existing
clusters/racks
Internet2 & NLR
geni
Exploring Networks
of the Future
Central PDC
at ISO
$t_{1,2}^{(k)}$
$t_{2,2}^{(k)}$
PDC2
$a_2^{(k+1)}$
$y_{21}(t)$
$y_{22}(t)$
PMUs
Area 2
$t_{2,4}^{(k)}$
$t_{1,4}^{(k)}$
PDC4
$a_4^{(k+1)}$
$y_{41}(t)$
$y_{42}(t)$
PMUs
Area 4

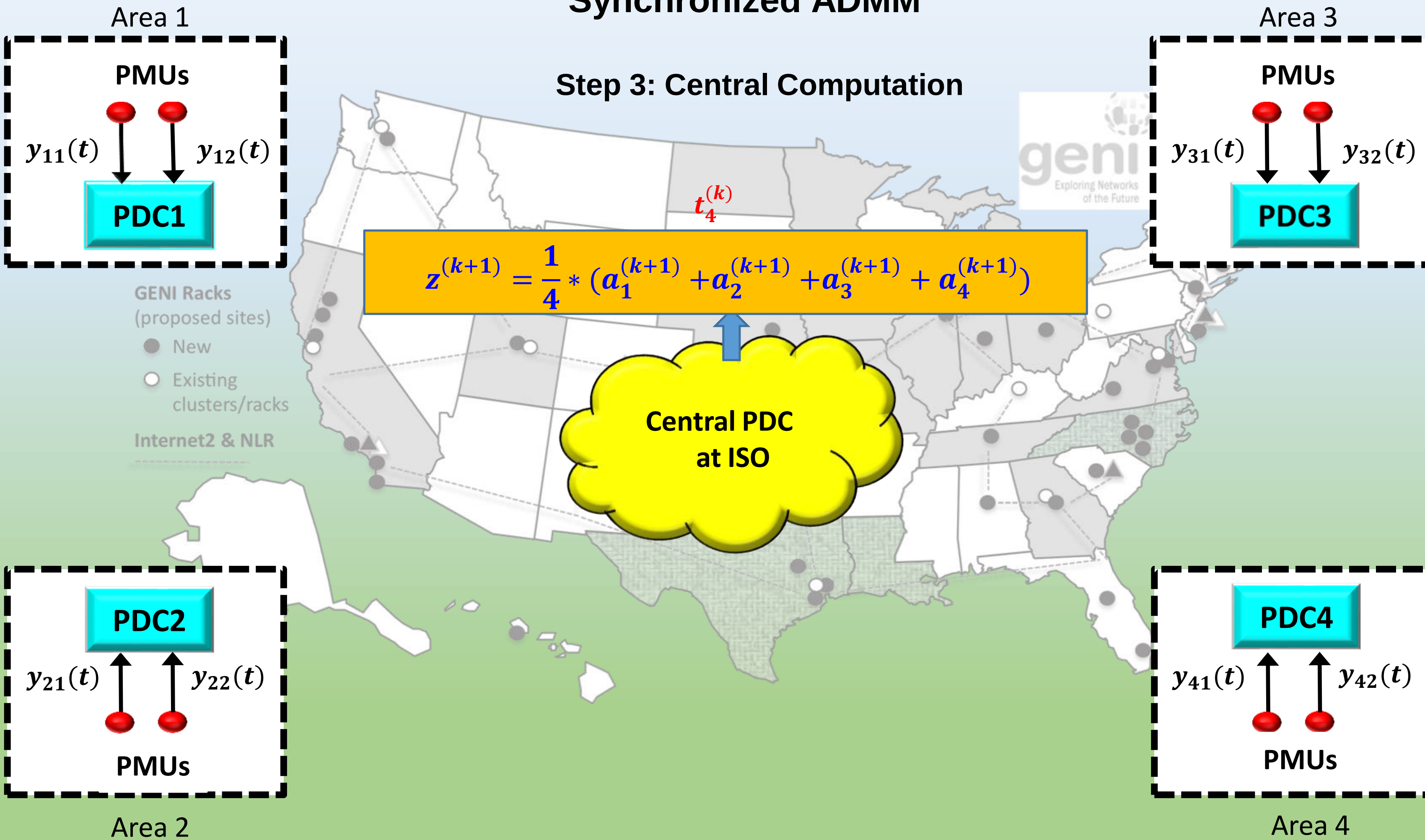
Synchronized ADMM

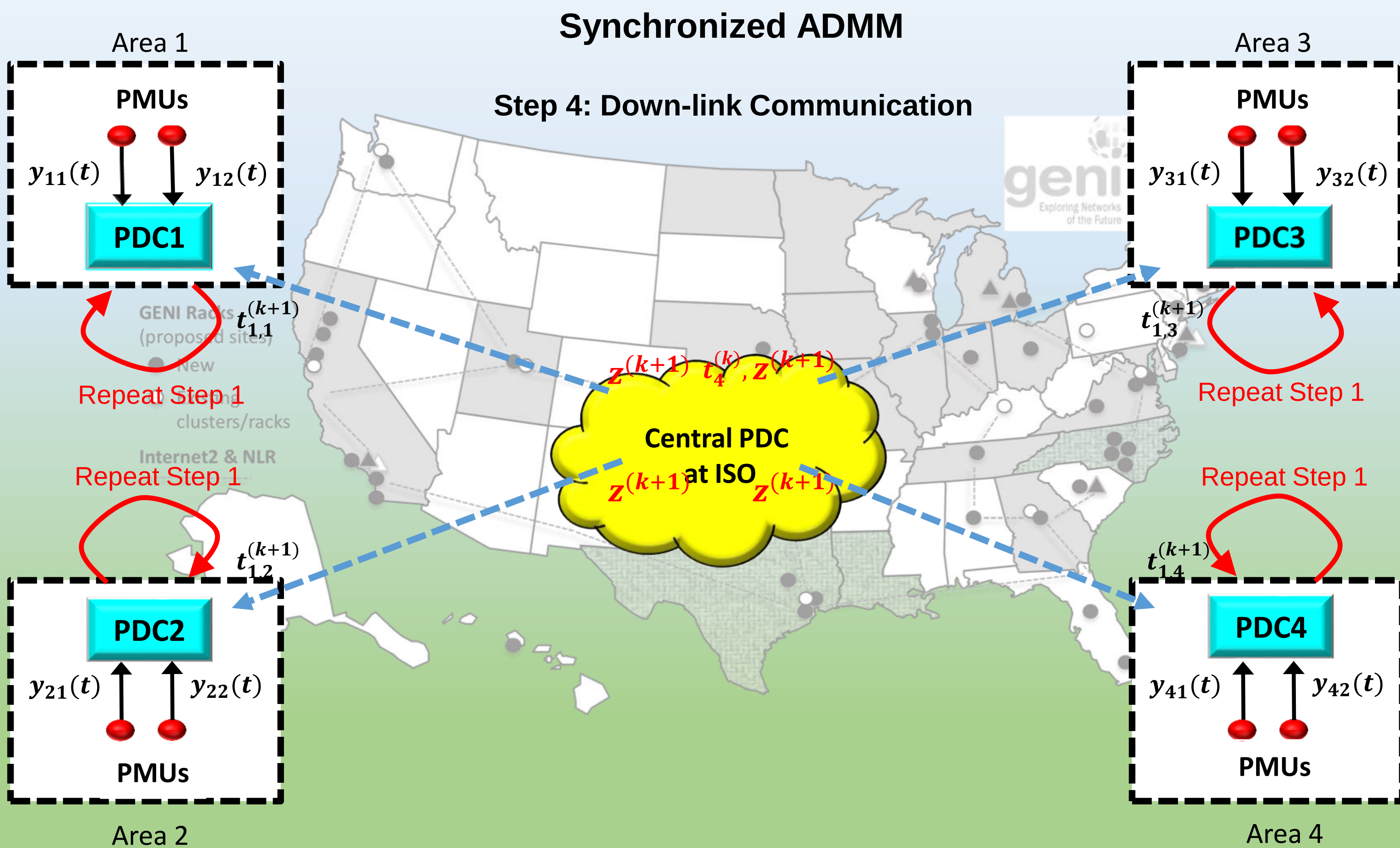
Step 2: Up-link Communication



Synchronized ADMM

Step 3: Central Computation





Synchronized ADMM
Step 4: Down-link Communication
Area 1
PMUs
$y_{11}(t)$
$y_{12}(t)$
PDC1
$t_{1,1}^{(k+1)}$
Repeat Step 1
Area 2
PDC2
$y_{21}(t)$
$y_{22}(t)$
PMUs
$t_{1,2}^{(k+1)}$
Repeat Step 1
Area 3
PMUs
$y_{31}(t)$
$y_{32}(t)$
PDC3
$t_{1,3}^{(k+1)}$
Repeat Step 1
Area 4
PDC4
$y_{41}(t)$
$y_{42}(t)$
PMUs
$t_{1,4}^{(k+1)}$
Repeat Step 1
Central PDC at ISO
$z^{(k+1)}$
$t_4^{(k)}, z^{(k+1)}$
$z^{(k+1)}$
$z^{(k+1)}$
GENI Racks (proposed sites)
clusters/racks
Internet2 & NLR
geni
Exploring Networks of the Future

Distributed Consensus Using ADMM

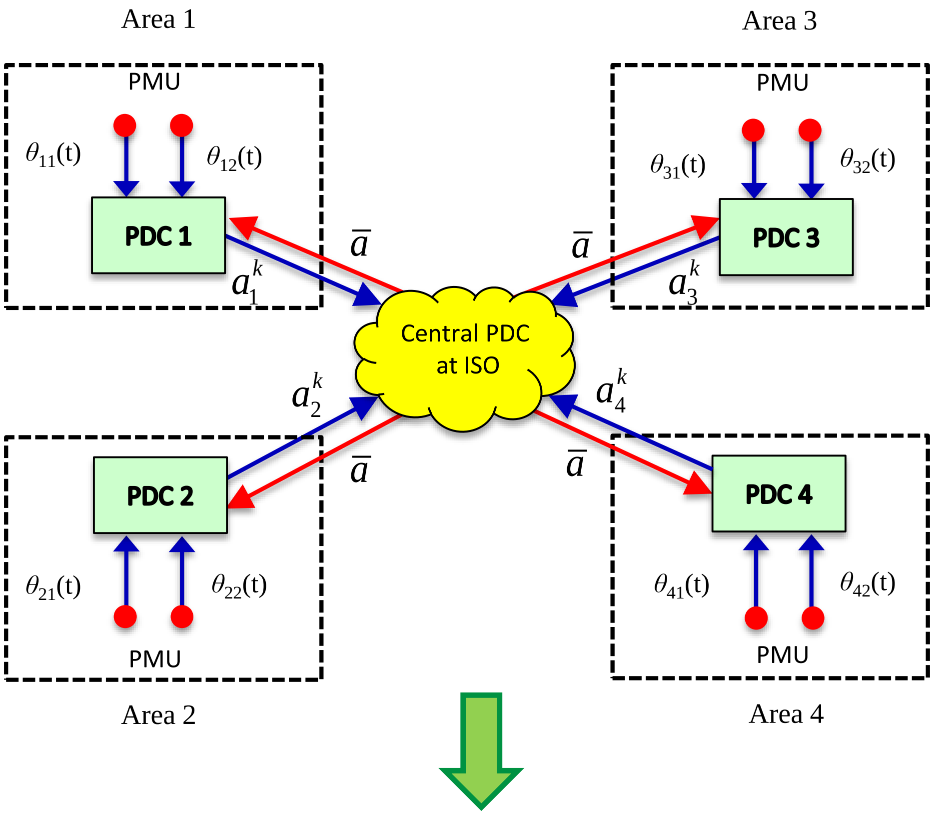
Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC *i*

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

- Step 2 Gather the values of $\mathbf{a}_i{}^{\boldsymbol{k+1}}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i{}^{\boldsymbol{k+1}}$
- Step 4 Broadcast the average value ($\bar{\mathbf{a}}_i{}^{\boldsymbol{k+1}}$) to local PDCs
- Step 5 Check the convergence
- Final Step Find the frequency Ω_i, and damping σ_i at each local PDC using $\bar{\mathbf{a}}_i{}^{\boldsymbol{k+1}}$



Privacy of PMU data between companies guaranteed

Distributed Consensus Using ADMM

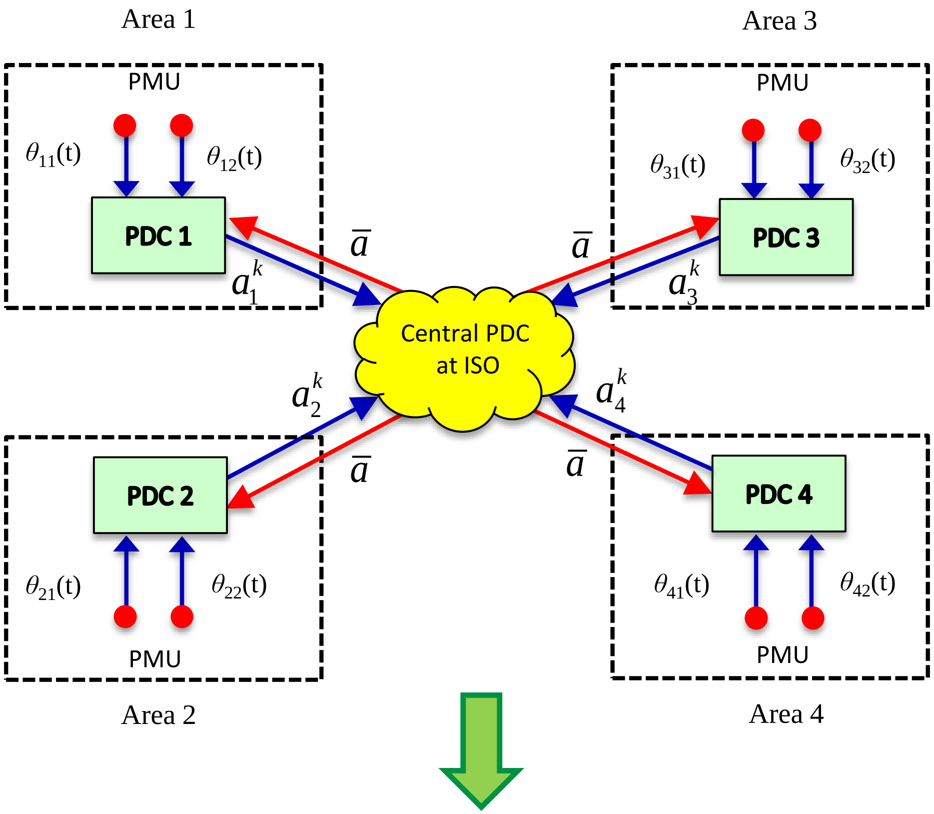
Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC i

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

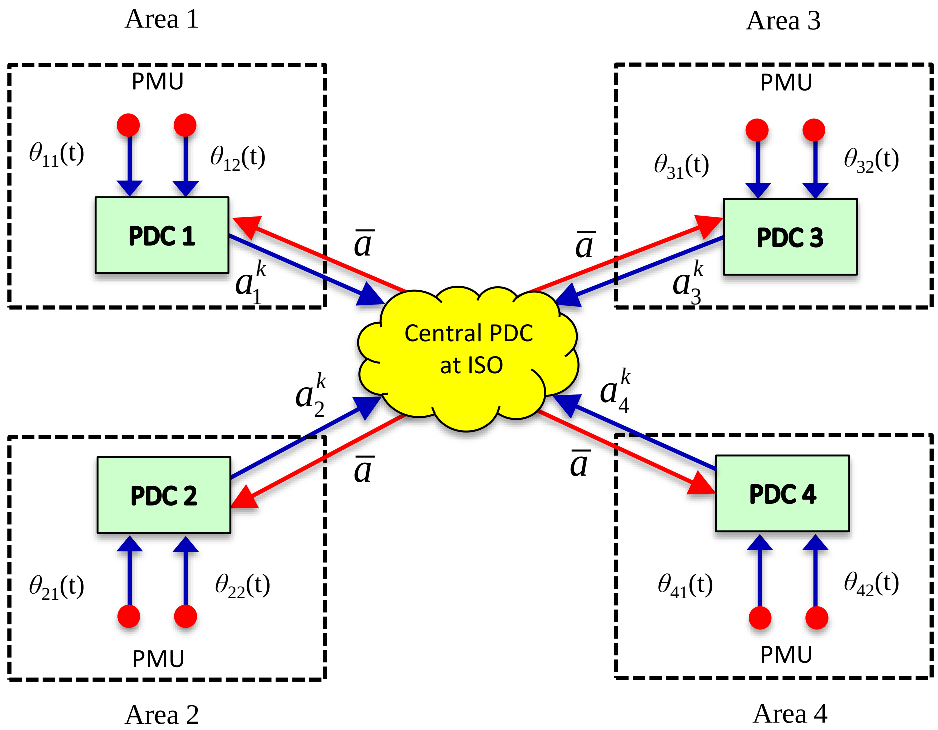
- Step 2 Gather the values of $\mathbf{a}_i^{k+1}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i^{k+1}$
- Step 4 Broadcast the average value $(\bar{\mathbf{a}}_i^{k+1})$ to local PDCs
- Step 5 Check the convergence
- Final Step Find the frequency Ω_i, and damping σ_i at each local PDC using $\bar{\mathbf{a}}_i^{k+1}$



Privacy of PMU data between companies guaranteed

- *perfect fit for differential privacy (Katewa, Chakrabortty, Gupta, ACC 2015)*

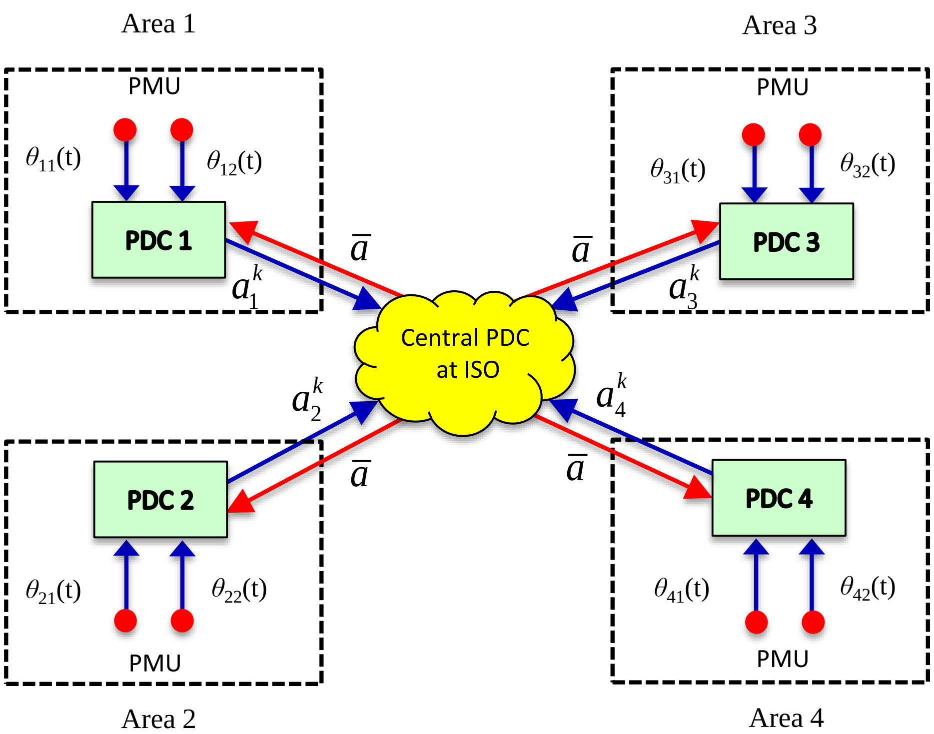
Cyber-Physical Coupling:
Incorporating Asynchronous Wide-Area Communication



Traffic Models for Internet Delays:

$$P(t) = \frac{1}{2}[\mathrm{erf}(\frac{\mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t-\mu}{\sqrt{2}\sigma})] + \frac{(1-p)}{N} e^{(\frac{1}{2}\lambda^2\sigma^2+\mu\lambda)} [\mathrm{erf}(\frac{\lambda\sigma^2+\mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t-\lambda\sigma^2-\mu}{\sqrt{2}\sigma})]$$

Cyber-Physical Coupling:
Incorporating Asynchronous Wide-Area Communication



If a message doesn't arrive at ISO by a delay threshold d_1*

- **Strategy 1:**

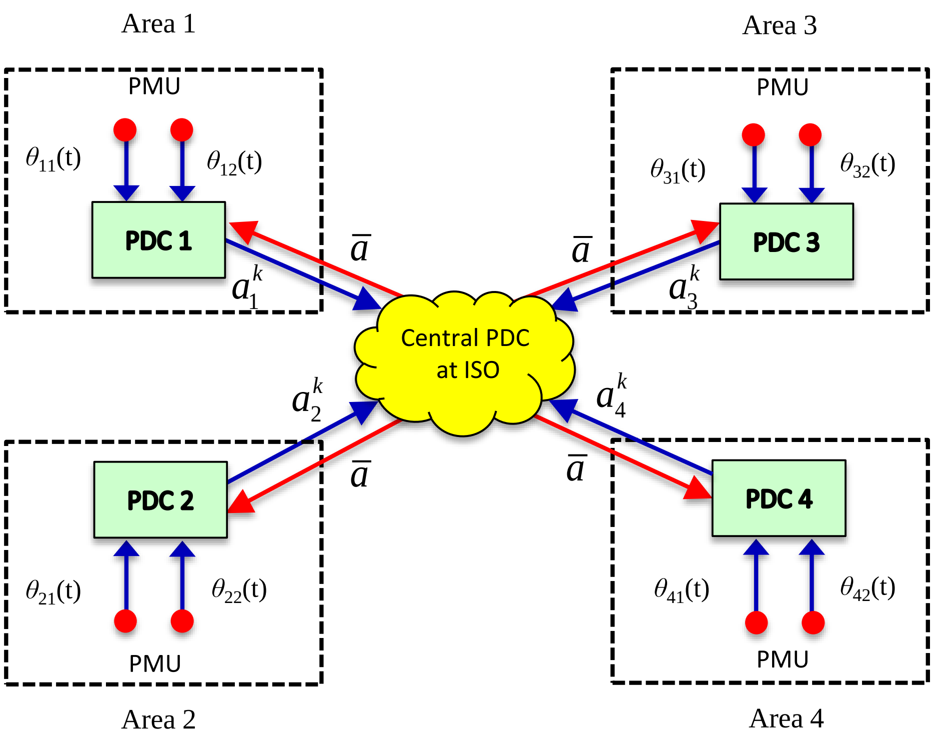
$$z^{(k+1)} = \frac{1}{|S_1^{(k)}|} \sum_{i \in S_1^{(k)}}^{N} \left(a_i^{(k+1)} + \frac{1}{\rho} w_i^{(k)}\right)$$

→ **Can easily lead to divergence**

Traffic Models for Internet Delays:

$$P(t) = \frac{1}{2}[\mathrm{erf}(\frac{\mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t-\mu}{\sqrt{2}\sigma})] + \frac{(1-p)}{N} e^{(\frac{1}{2}\lambda^2\sigma^2 + \mu\lambda)} [\mathrm{erf}(\frac{\lambda\sigma^2 + \mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t - \lambda\sigma^2 - \mu}{\sqrt{2}\sigma})]$$

Cyber-Physical Coupling:
Incorporating Asynchronous Wide-Area Communication



If a message doesn't arrive at ISO by a delay threshold d_1*

- **Strategy 2:**

$$z^{(k+1)} = \frac{1}{N}\left(\sum_{i\in S_1^{(k)}}^{N}(a_i^{(k+1)} + \frac{1}{\rho}w_i^{(k)}) + \sum_{i\notin S_1^{(k)}}^{N}(a_i^{(k)} + \frac{1}{\rho}w_i^{(k-1)})\right)$$

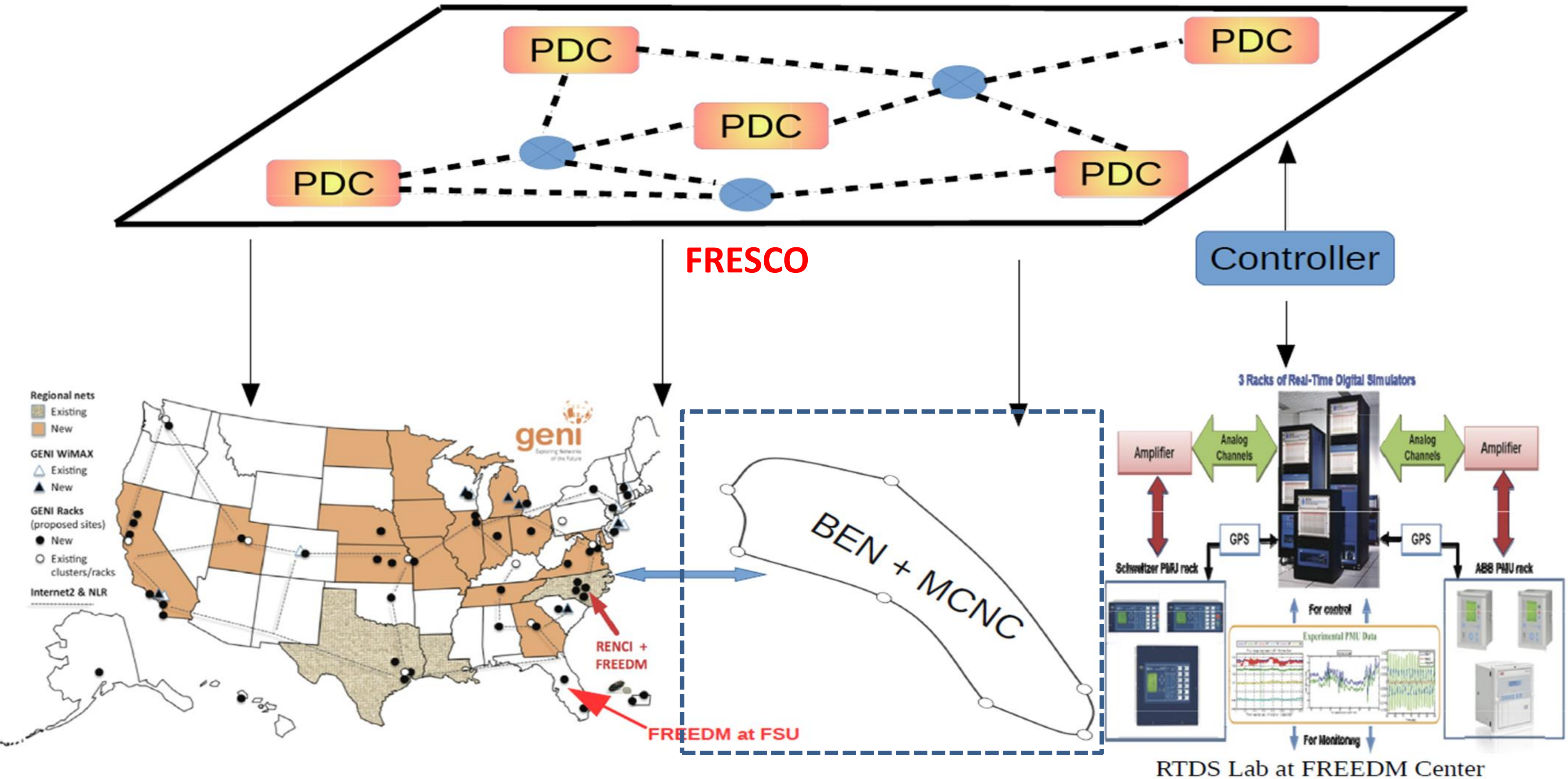
Substitute values from previous iteration

Convergent, but slow

Traffic Models for Internet Delays:

$$P(t) = \frac{1}{2}[\text{erf}(\frac{\mu}{\sqrt{2}\sigma}) + \text{erf}(\frac{t-\mu}{\sqrt{2}\sigma})] + \frac{(1-p)}{N}e^{(\frac{1}{2}\lambda^2\sigma^2+\mu\lambda)}[\text{erf}(\frac{\lambda\sigma^2+\mu}{\sqrt{2}\sigma}) + \text{erf}(\frac{t-\lambda\sigma^2-\mu}{\sqrt{2}\sigma})]$$

ExoGENI-WAMS Testbed at NC State & RENCI/UNC Chapel Hill



Middleware provided by Green Energy Corporation and RTI

Announcement # 1

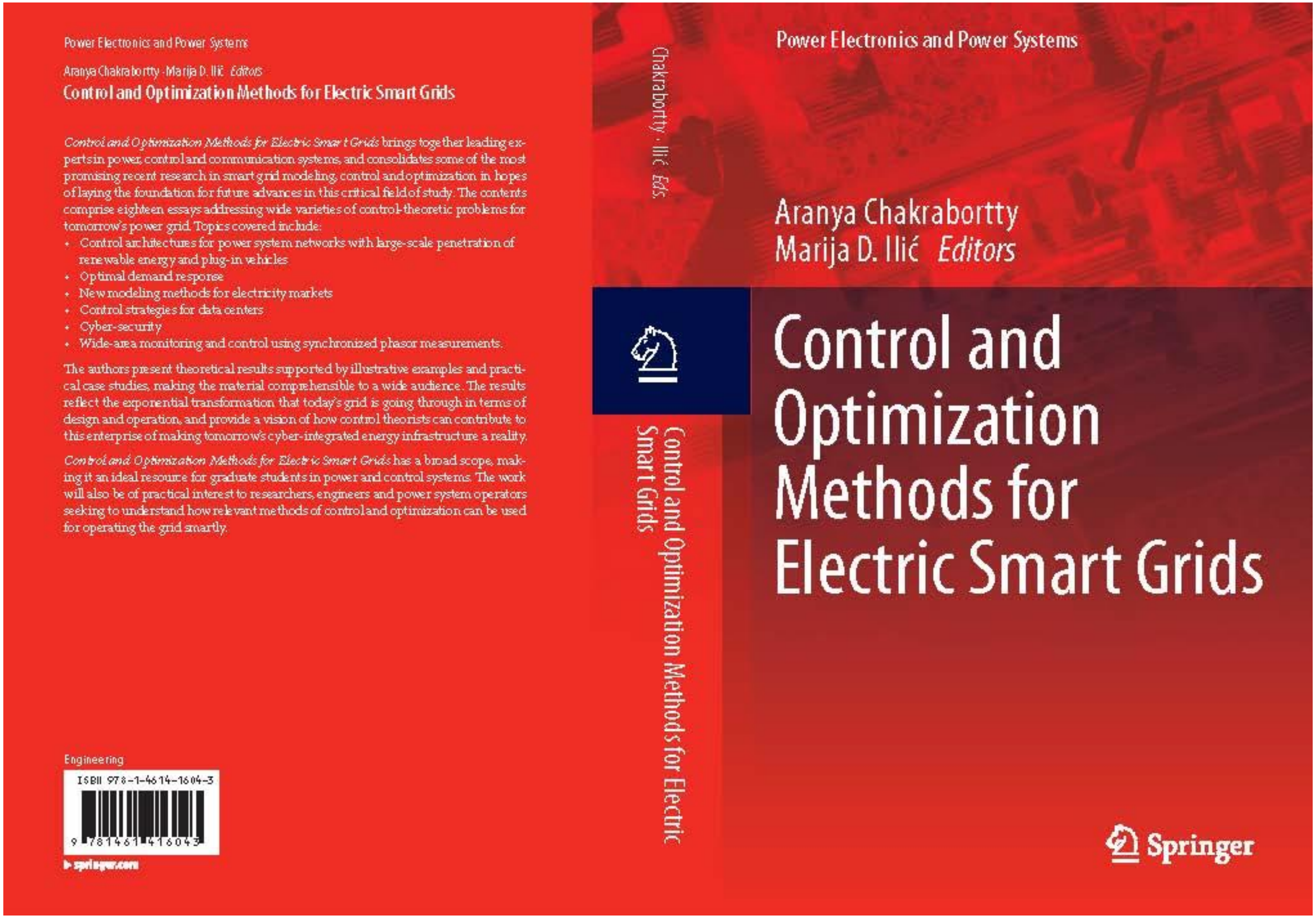
American Control Conference 2016

July 6-8, 2016 in Boston, MA

- Chair for Industry and Applications & Tutorials

- please email me if you want to organize a tutorial or special session!

Announcement # 2



Thank You

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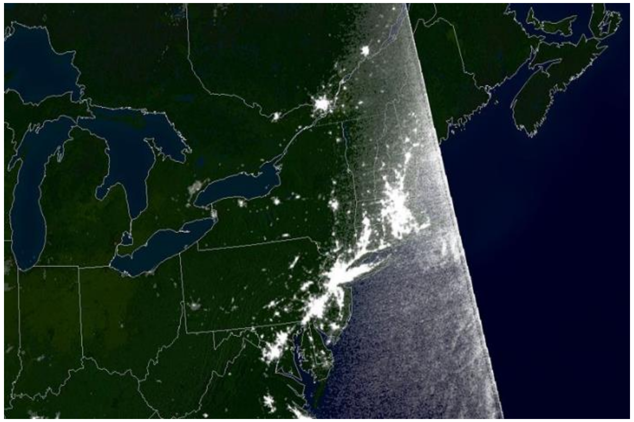
Conclusions

1. WAMS is a tremendously promising technology for control researchers
2. Control + Communications + Computing (CPS) must merge
3. Plenty of new research problems – EE, Applied Math, Computer Science
4. Plenty of new distributed optimization and control problems
5. Both theory and testbed experiments must progress
6. Right time to think mathematically – Network theory is imperative electric grid
7. Needs participation of young researchers!
8. Promises to create jobs and provide impetus to power engineering

Main trigger: 2003 Northeast Blackout

NYC before blackout

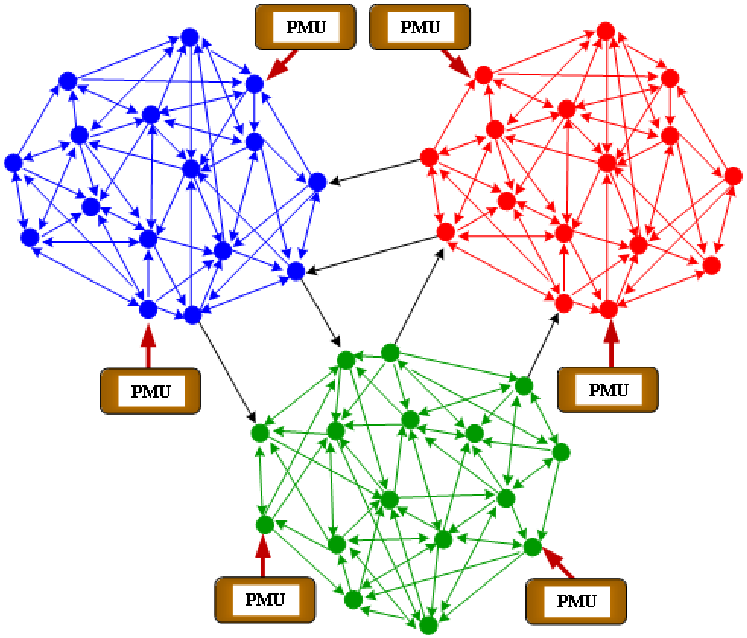


NYC after blackout

2 Main Lessons Learnt from the 2003 Blackout:

1. Need significantly higher resolution measurements

⇨ From traditional SCADA (System Control and Data Acquisition) to PMUs (Phasor Measurement Units)

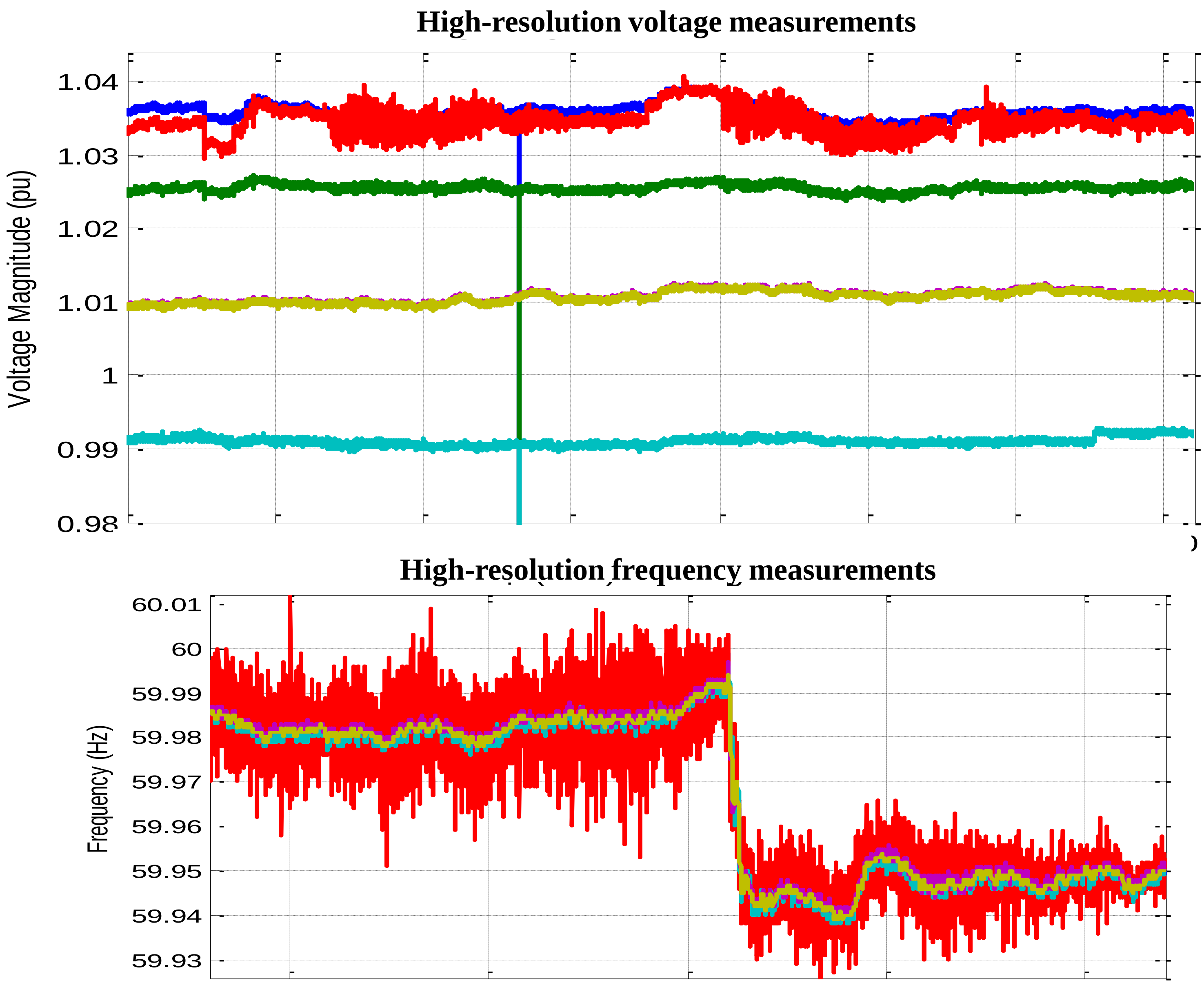


2. Local monitoring & control can lead to disastrous results

⇨ Coordinated control instead of selfish control

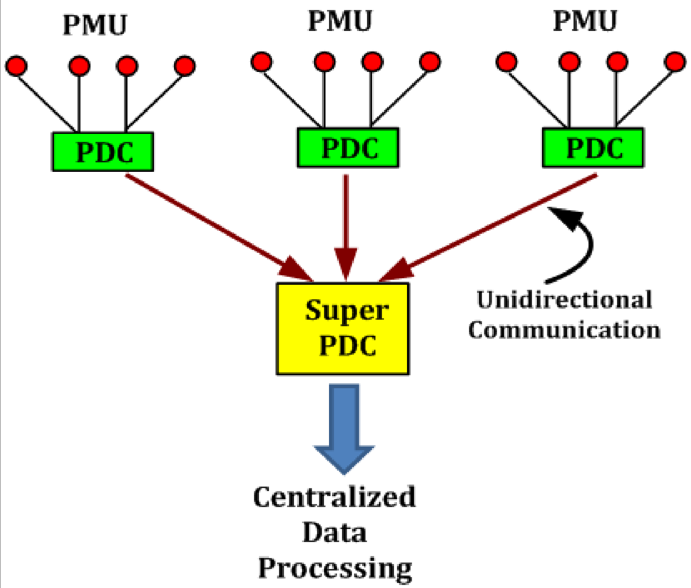
Hauer, Zhou & Trudnowsky, 2004
Kosterev & Martins, 2004

High-resolution PMU measurements from the US west coast grid



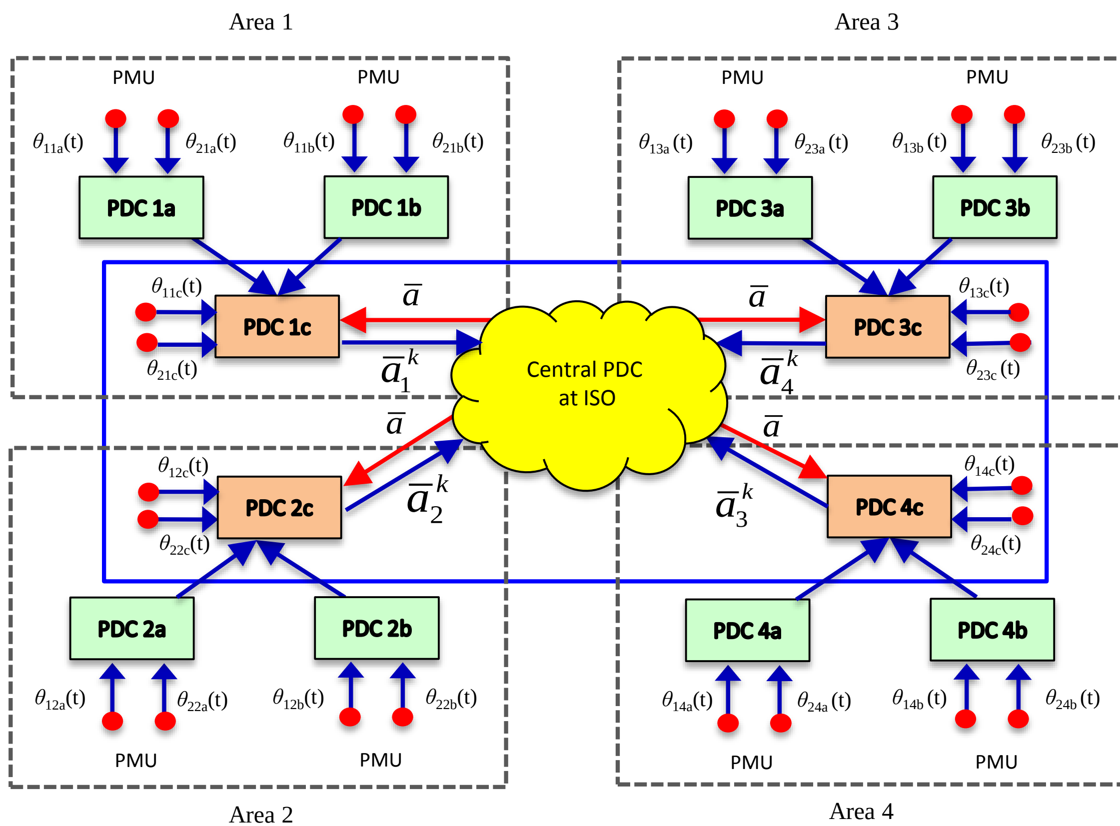
Centralized vs Distributed Algorithms

Centralized RLS



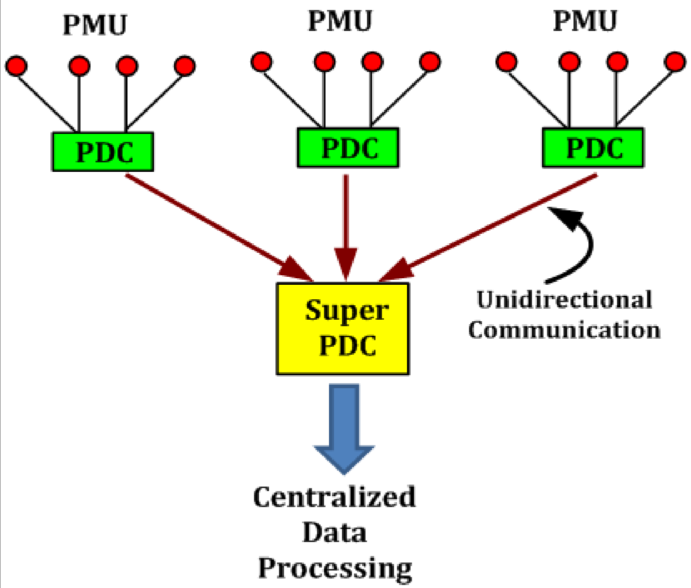
Control Room

Heirarchically Distributed Prony

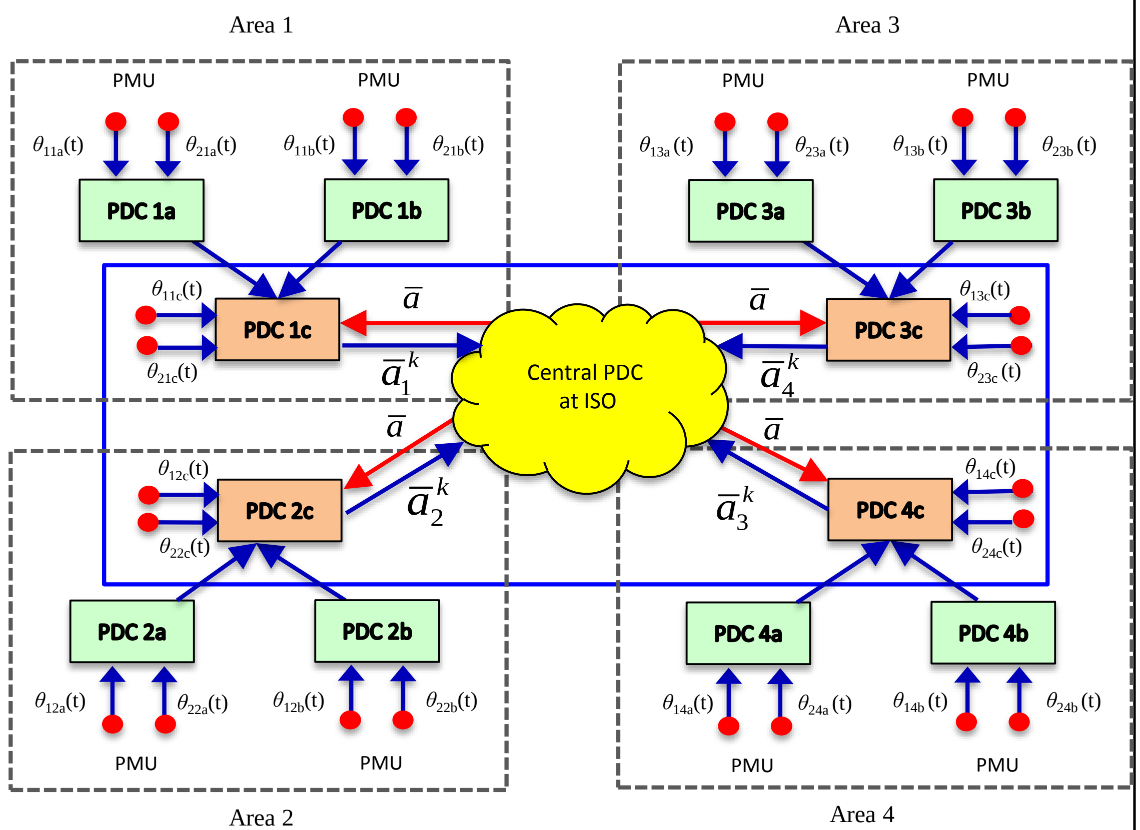


Centralized vs Distributed Algorithms

Centralized RLS



Heirarchically Distributed Prony



Control Room

Specific application of our interest:

Wide-area oscillation monitoring

Motivating the Wide-Area Oscillation Monitoring Problem:

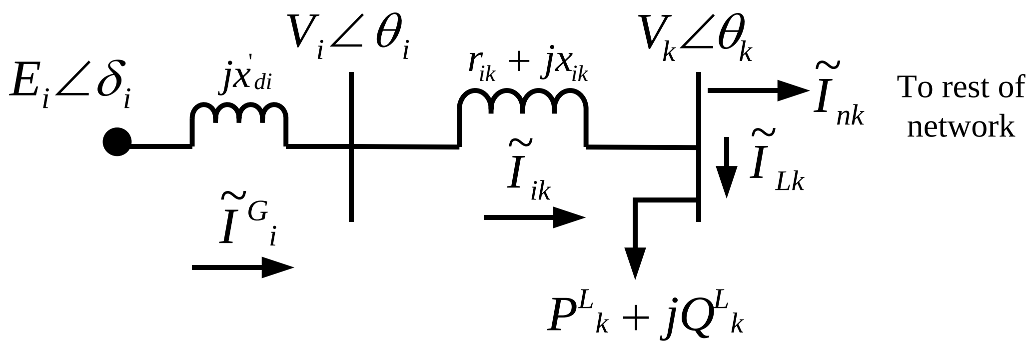
• Synchronous Generator Models

$$\dot{\delta}_i = \omega_i - \omega_s$$
$$M_i \dot{\omega}_i = P_{mi} - D_i(\omega_i - \omega_s) - P_i^G$$
$$\tau_i \dot{E}_i = -\frac{x_{di}}{x'_{di}} E_i + \frac{x_{di} - x'_{di}}{x'_{di}} V_i \cos(\delta_i - \theta_i) + E_{Fi}$$

$E_{Fi} = \bar{E}_{Fi} + E_i$

Control input
Excitation voltage

• Power Flow Equations



$$P_i^G = \frac{E_i V_i}{x'_d i} \sin(\delta_i - \theta_i) + \left(\frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}}\right) V_i^2 \sin(2(\delta_i - \theta_i))$$
$$Q_i^G = \frac{E_i V_i}{x'_d i} \cos(\delta_i - \theta_i) - \left(\frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}} - \frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}} \cos(2(\delta_i - \theta_i))\right) V_i^2,$$

Bus voltage and phase angle
Algebraic variables
Measured by PMU

Grid Dynamic Models

• **Load Models**

$$P_j^L = a_j V_j^2 + b_j V_j + c_j$$
$$Q_j^L = e_j V_j^2 + f_j V_j + g_j$$

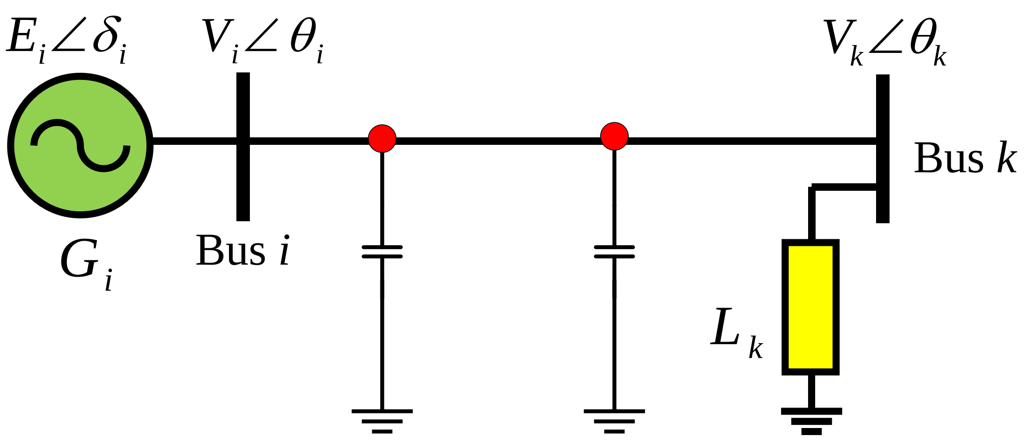
$a_j, e_j =$ constant impedance
$b_j, f_j =$ constant current
$c_j, g_j =$ constant power

• **Transmission Line Model**

$$P_{ij} = G_{ij} V_i^2 + B_{ij} V_i V_j \sin(\theta_i - \theta_j) - G_{ij} V_i V_j \cos(\theta_i - \theta_j)$$

$$Q_{ij} = (B_{ij} - B_{ij}^c) V_i^2 - B_{ij} V_i V_j \cos(\theta_i - \theta_j) - G_{ij} V_i V_j \sin(\theta_i - \theta_j).$$

Pi-model

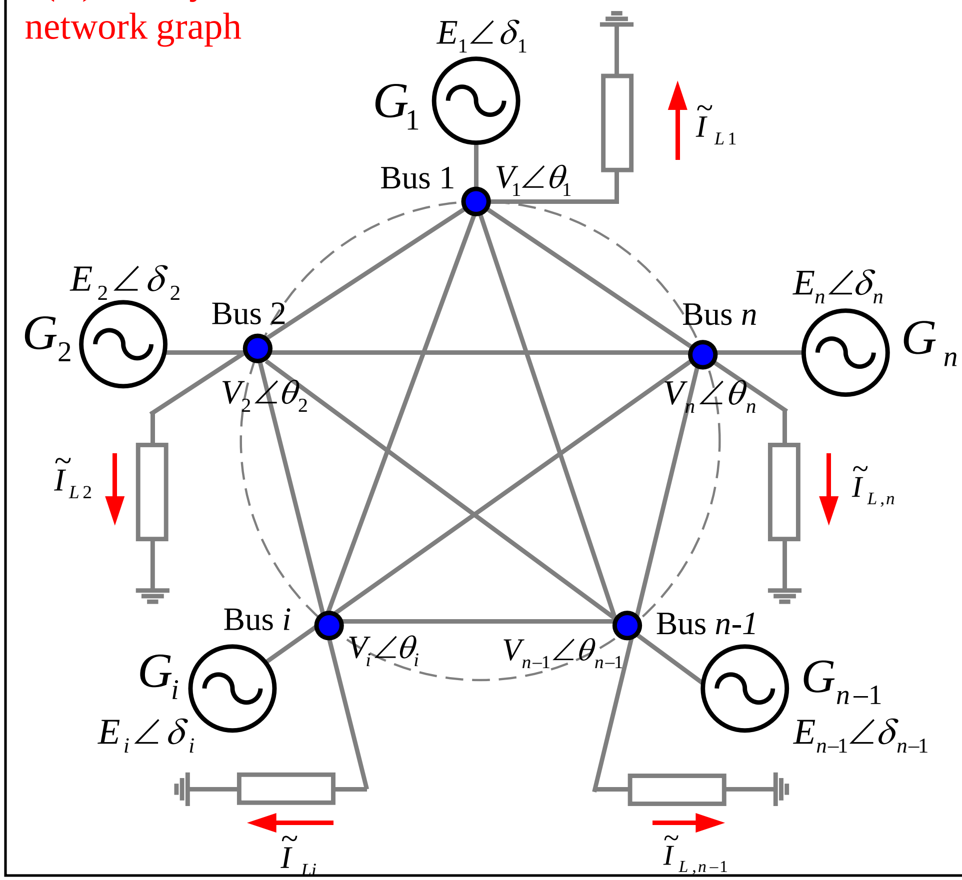


• Total Network Model

$$\begin{bmatrix}\Delta\dot{\delta}\\ M\Delta\dot{\omega}\\ \Delta\dot{E}\end{bmatrix} = \begin{bmatrix}0 & I & 0\\ -L(G) & -D & -P\\ K & 0 & J\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\\ \Delta E\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \mathrm{col}_{i=1(1)n}(\gamma_i)\\ \mathrm{col}_{i=1(1)n}(\rho_i)\end{bmatrix}}_{\text{due to load}} + \begin{bmatrix}0 & 0\\ 0 & I\\ I & 0\end{bmatrix}\begin{bmatrix}\Delta P_m\\ \Delta E_F\end{bmatrix} \quad \ldots(1)$$

$L(G)$ = fully connected network graph

Controllable inputs

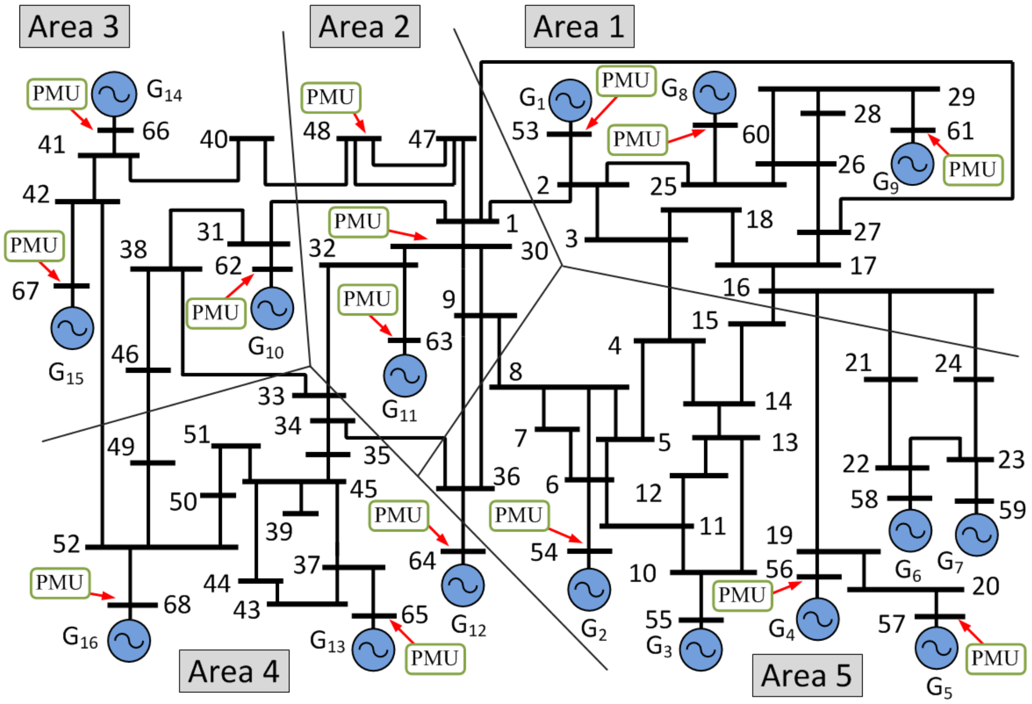


Output Equation

$$y = \mathrm{col}_{i\in S}(\Delta V_i,\ \Delta\theta_i). \qquad \ldots(2)$$

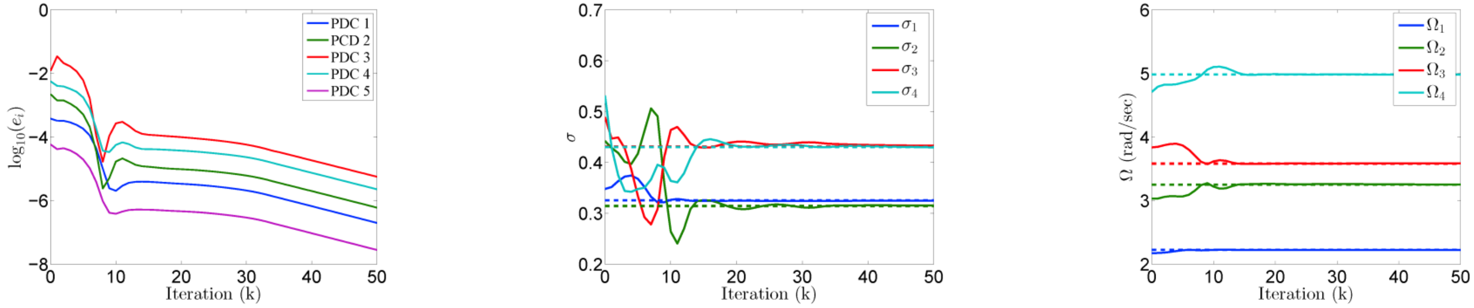
Simulation Results

IEEE-68 Bus Model (simplified model of the New-England power system)

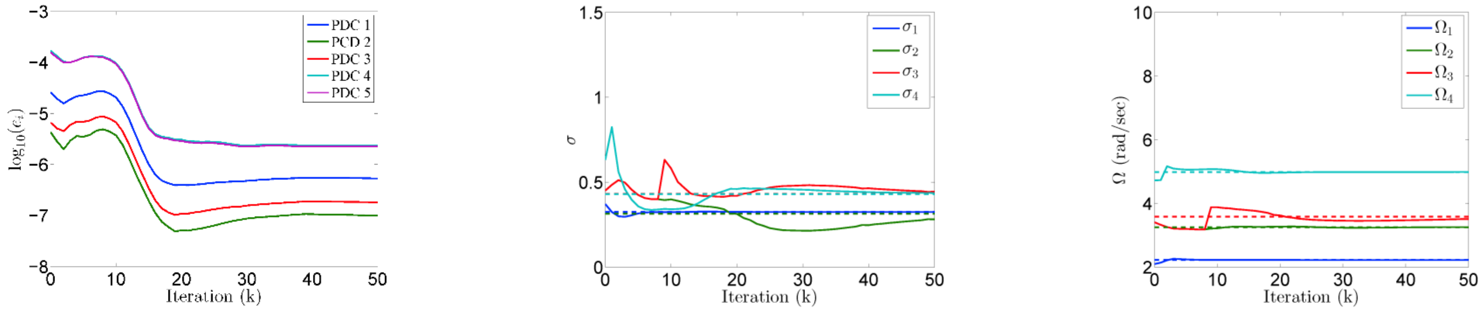


- 68 Bus, 16 Generators
- 5 Computational Areas
- Simulations are performed in Power System Toolbox (PST)
- A three-phase fault occurred at line connecting buses 1 and 2, started at t=0.1 (sec), cleared at bus 1 at t=0.15 (sec), and cleared at bus 2 at t=0.2 (sec).

Distributed Prony:

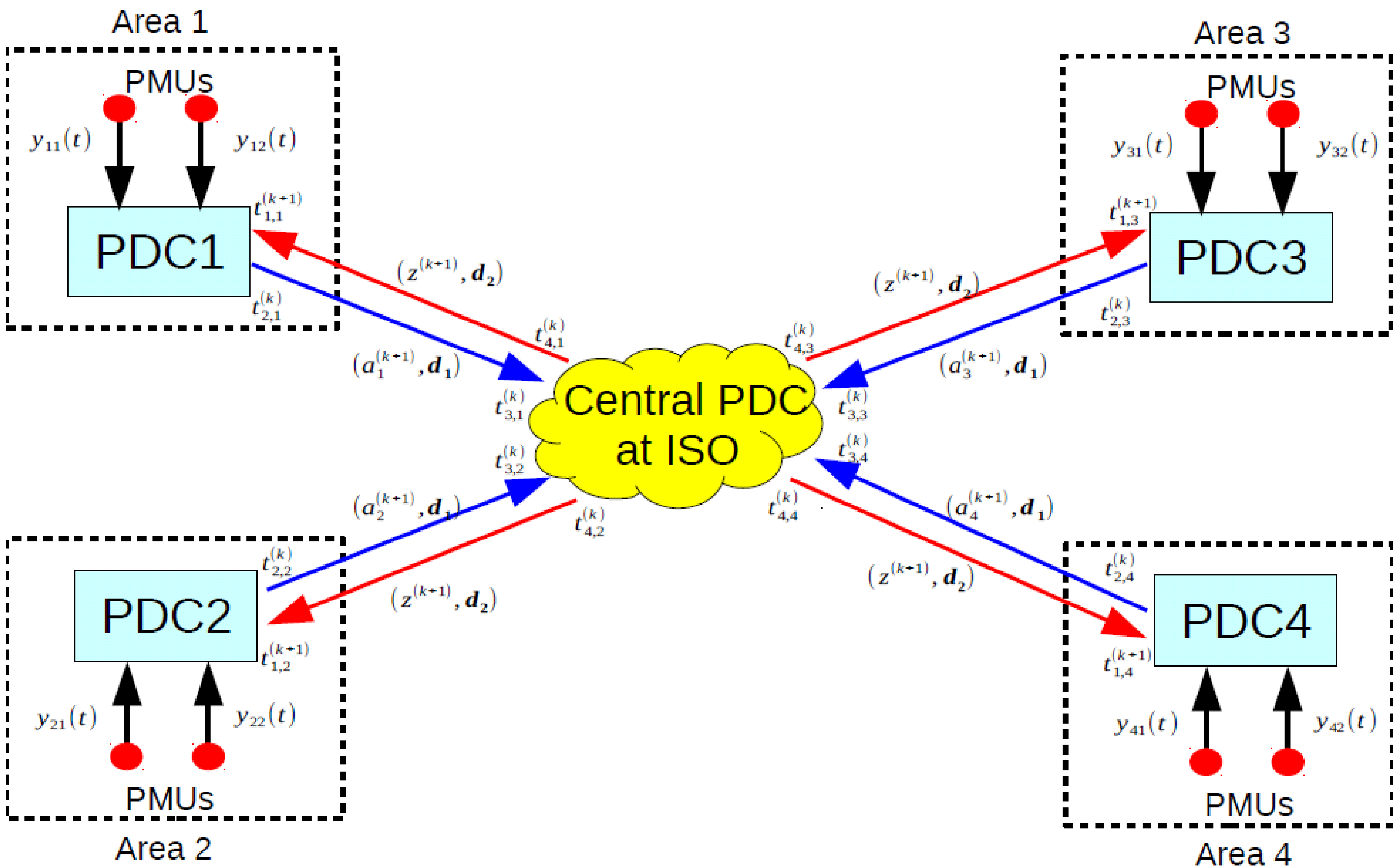


In Case of Communication Failure (1 healthy communication link in 10 iterations)



Actual value	Centralized Prony	Diistributed Prony	Distributed Prony with Comm Failure
-0.3256□j2.2262	-0.3250□j2.2230	-0.3247□j2.2230	-0.3243□j2.2225
-0.3143□j3.2505	-0.3146□j3.2531	-0.3153□j3.2525	-0.2808□j3.2560
-0.4312□j3.5809	-0.4318□j3.5849	-0.4328□j3.5855	-0.4443□j3.5106
-0.4301□j4.9836	-0.4308□j4.9865	-0.4294□j4.9798	-0.4361□j4.9853

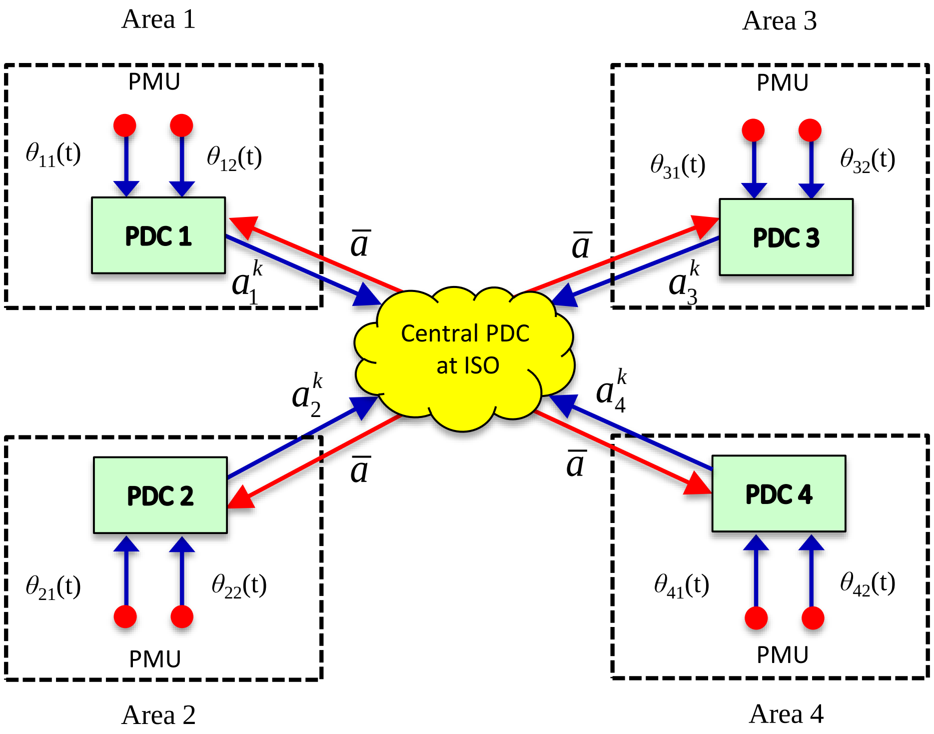
Incorporating Asynchronous Communication



Distributed Consensus Using ADMM

Iteration 0

Initialize the primal variable $\mathbf{a}_i^{\mathbf{0}}$ and the dual variable $\mathbf{w}_i^{\mathbf{0}}$ at each local PDC i



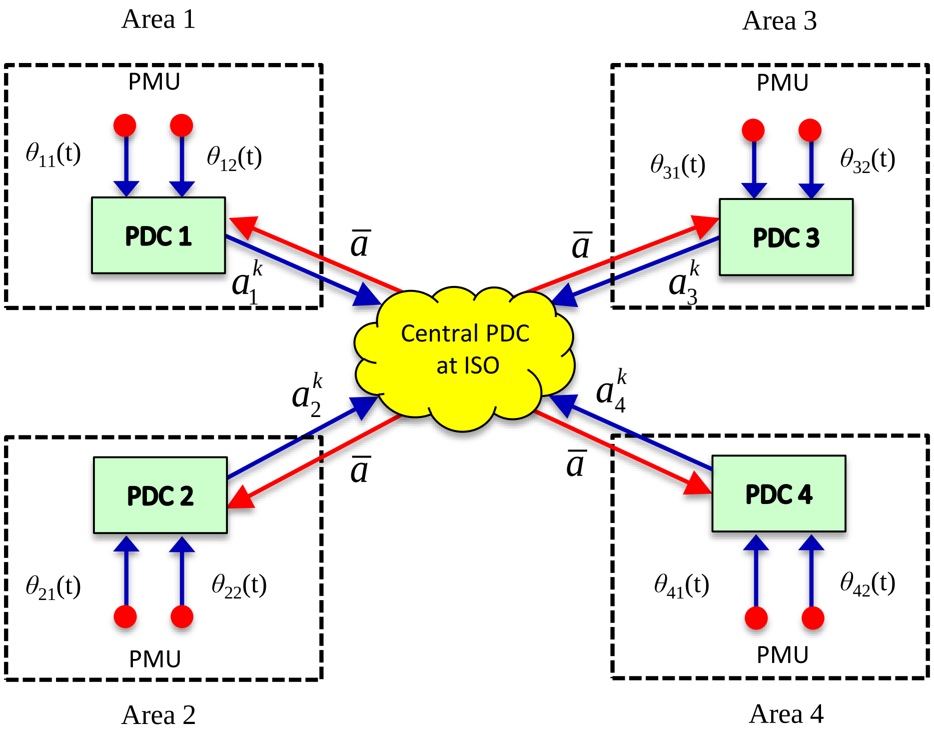
Distributed Consensus Using ADMM

Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC i

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$



Distributed Consensus Using ADMM

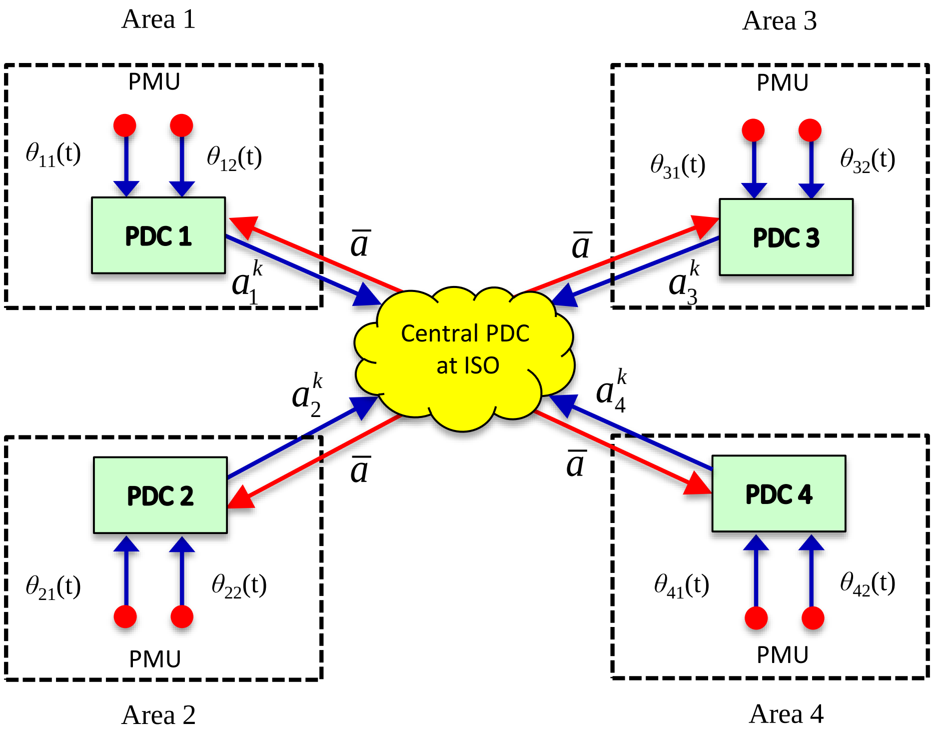
Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC *i*

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$
$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

- Step 2 Gather the values of $\mathbf{a}_i^{k+1}$ at the central PDC

- Step 3 Take the average of $\mathbf{a}_i^{k+1}$



Distributed Consensus Using ADMM

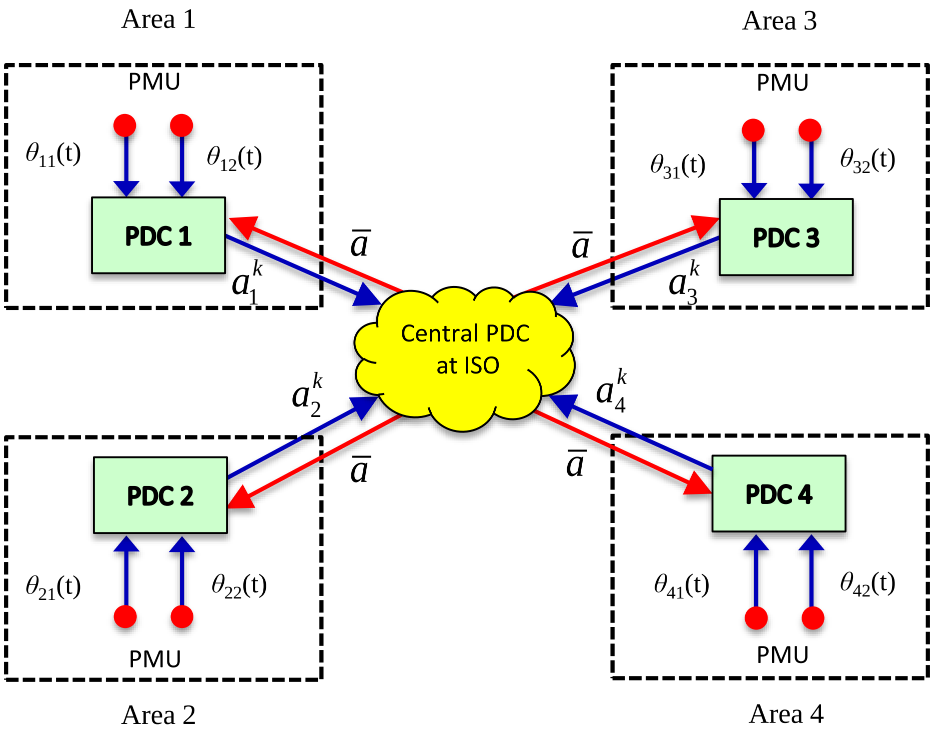
Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC *i*

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

- Step 2 Gather the values of $\mathbf{a}_i{}^{k+1}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i{}^{k+1}$
- Step 4 Broadcast the average value $(\overline{\mathbf{a}_i}{}^{k+1})$ to local PDCs
- Step 5 Check the convergence



Distributed Consensus Using ADMM

Iteration *k+1*

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC i

$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

- Step 2 Gather the values of $\mathbf{a}_i^{k+1}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i^{k+1}$
- Step 4 Broadcast the average value $(\bar{\mathbf{a}}_i^{k+1})$ to local PDCs
- Step 5 Check the convergence
- Final Step Find the frequency Ω_i, and damping σ_i at each local PDC using $\bar{\mathbf{a}}_i^{k+1}$