# Demand Response with Linear Bidding: Efficiency vs. Risk

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# Collaboration

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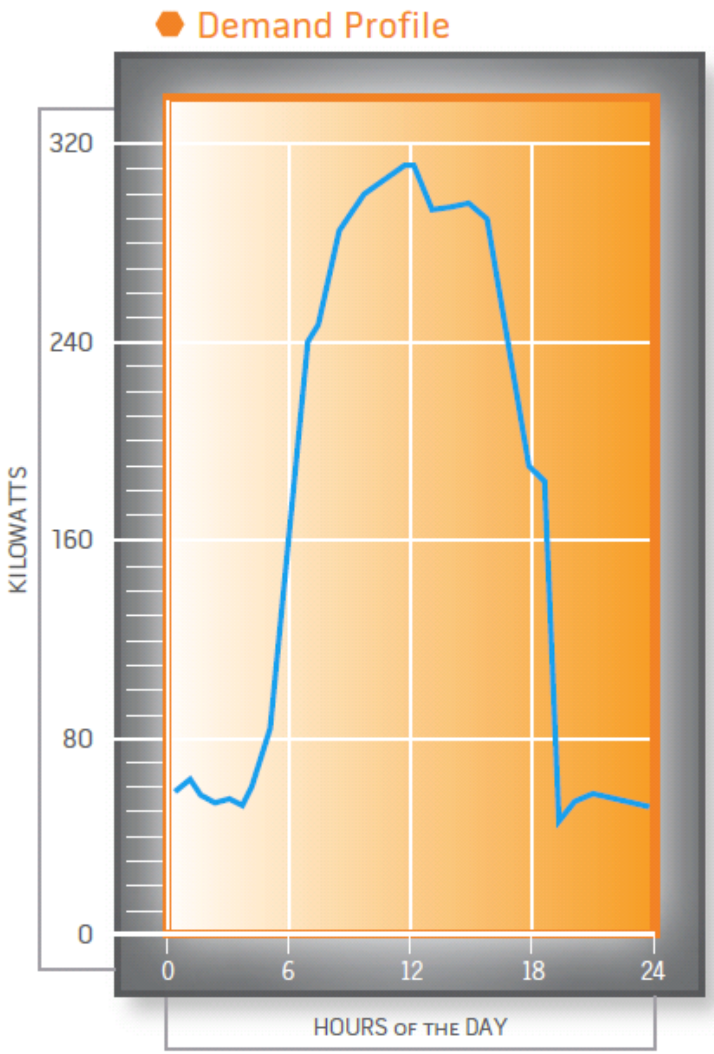
# Demand Response: Demand

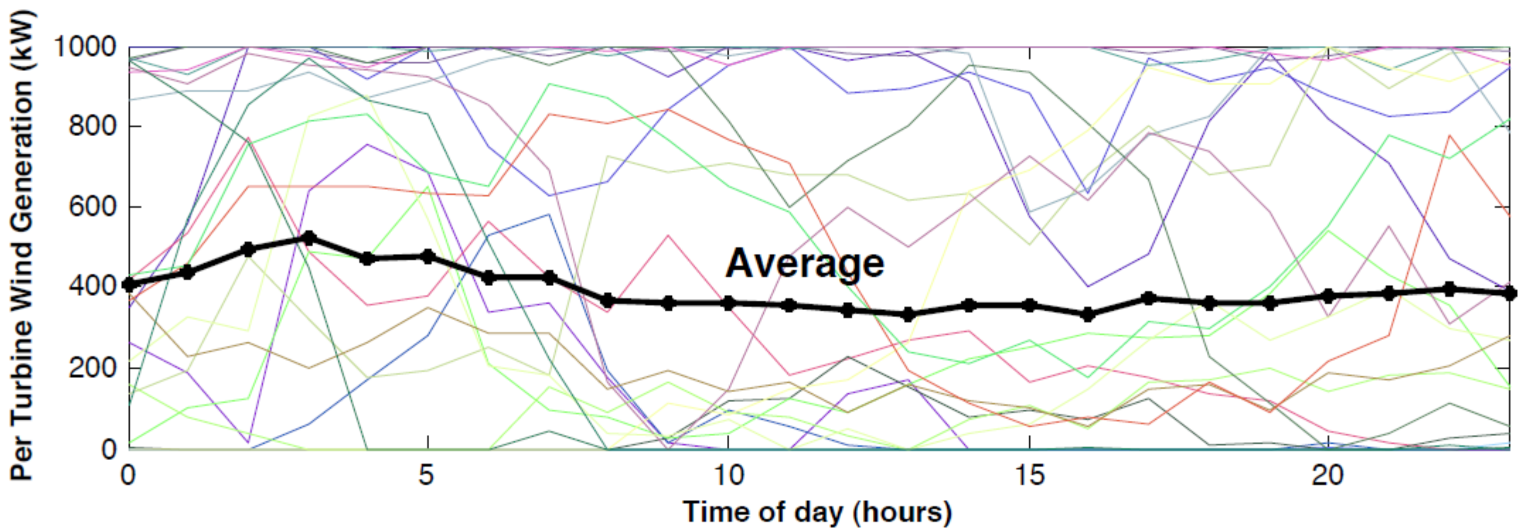



- Electricity demand: highly time-varying
- Provision for peak load
  - Low load factor
    - National load factor is about 55%
  - Underutilized
    - 10% of generation and 25% of distribution facilities are used less than 5% of the time
- A way out: Shape the demand
  - Reduce the peak
  - Smooth the variation

Source: DoE, Smart Grid Intro, 2008

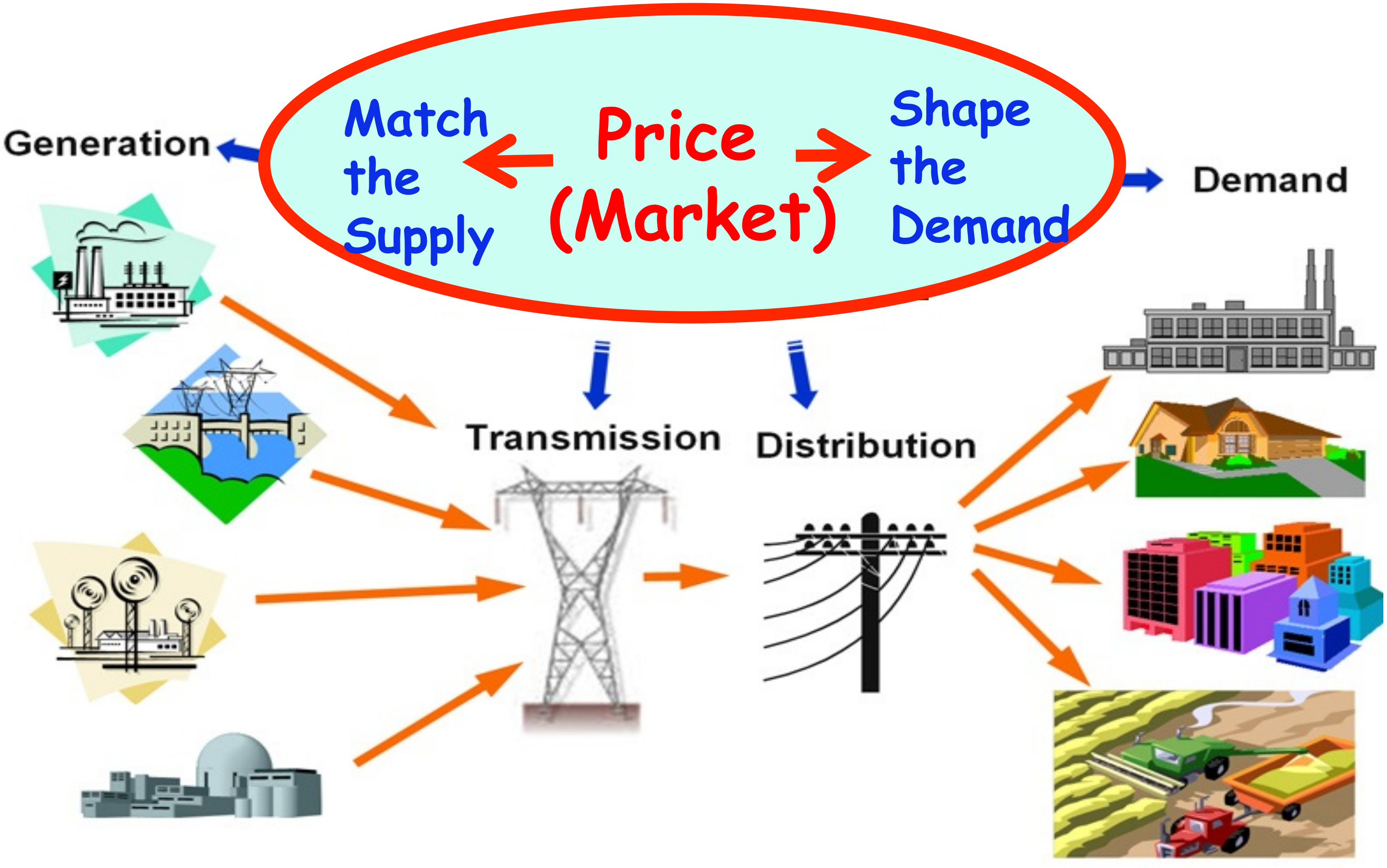
# Demand Response: Generation

- Supply becomes highly time-varying
  - steady rise of renewable energy resources
    - Intermittent generation
  - Large storage is not available
- A way out: Match the supply ⇦ This work



# Demand Response

Use incentive mechanisms such as real-time pricing to induce customers to shift usage or reduce (even increase) consumption



# Overall structure

generation


wholesale market

**utility company**

retail market

customer

# Main issues

## The role of utility as an intermediary

- Play in multiple wholesale markets to provision aggregate power to meet demands
  - day-ahead, real-time balancing, ancillary services
- Resell, with appropriate pricing, to the end users
- Provide two important values
  - Aggregate demand at the wholesale level so that overall system is more efficient
  - Absorb large uncertainty/complexity in wholesale markets and translate them into a smoother environment (both in prices and supply) for the end users.

How to quantify these values and price them in the form of appropriate contracts/pricing schemes?

# Main issues

Utility/end users interaction

- Design objective
  - Welfare-maximizing, profit-maximizing
- Price-taking (Competitive) vs Price-anticipating (Game)
- Price of Anarchy
- Risk assessment (possible value of Anarchy)

# The basics of supply and demand

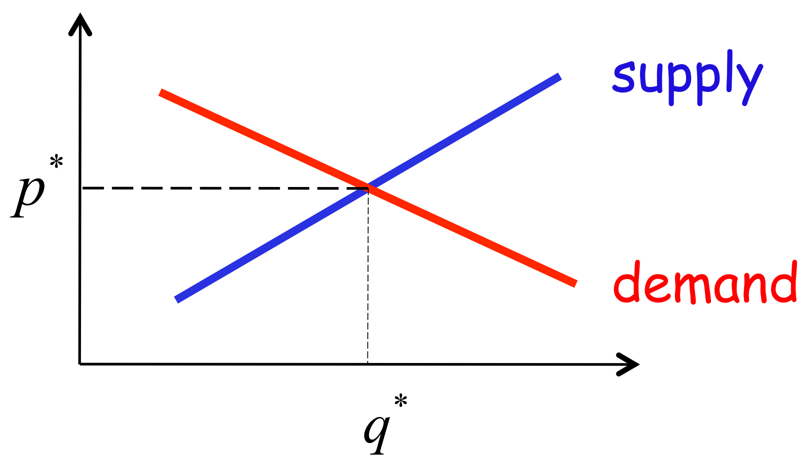
- Supply function: quantity demanded at given prices

$$q = S(p)$$

- Demand function: quantity supplied at given prices

$$q = D(p)$$

- Market equilibrium: $(q^*, p^*)$ such that $q^* = S(p^*) = D(p^*)$
  - No surplus, no shortage, price clears the market



# Problem setting

- Supply deficit (or surplus) on electricity: $d$

  weather change, unexpected events, …
- Supply is inelastic

**Problem**: How to allocate the deficit among demand-responsive customers?

# Supply function bidding

- Customer $i$ load to shed: $q_i$
- Customer $i$ supply function (SF):

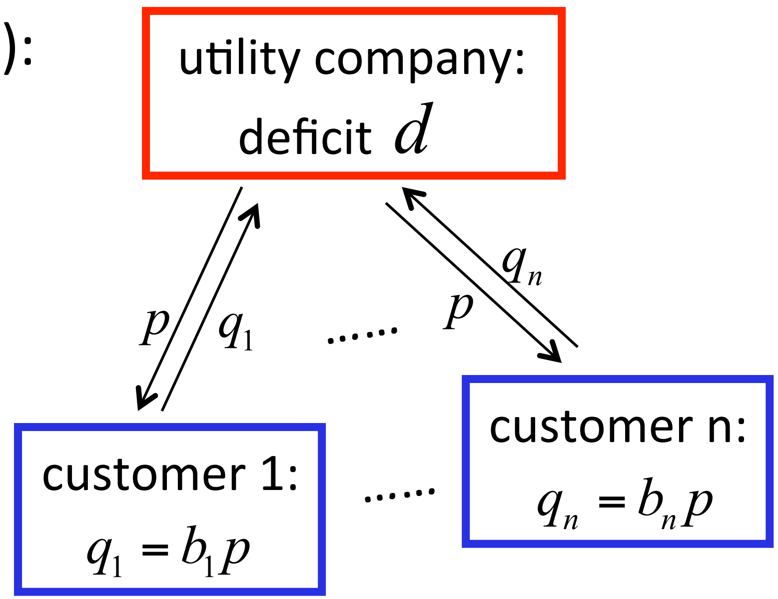
$$q_i(b_i, p) = b_i p$$

  - the amount of load that the customer is committed to shed given price $p$

- Market-clearing pricing:

$$\sum_i q_i(b_i, p) = d$$

$$p = p(b) @ d / \sum_i b_i$$



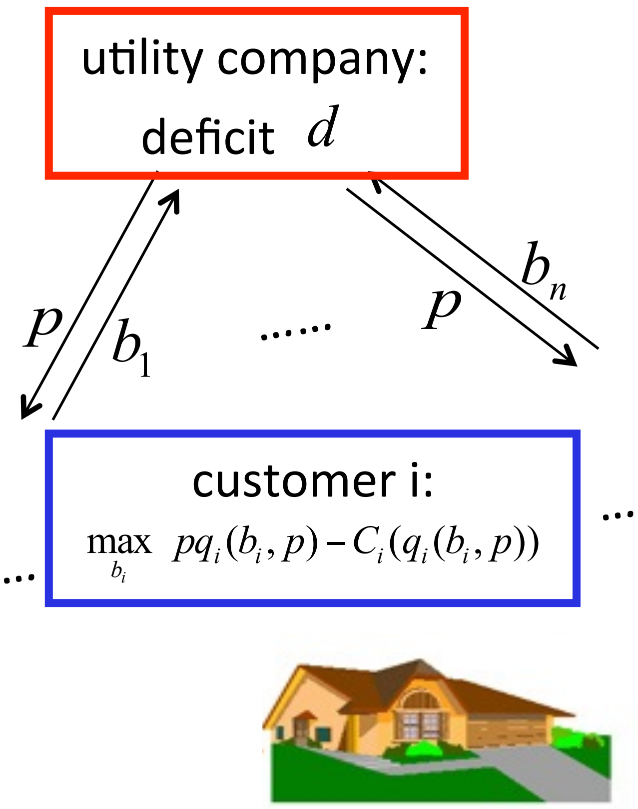

# Parameterized supply function

- Adapts better to changing market conditions than does a simple commitment to a fixed price of quantity (Klemper & Meyer ’89)
  - widely used in the analysis of the wholesale electricity markets
  - Green & Newbery ‘92, Rudkevich et al ‘98, Baldick et al ‘02, ‘04, …
- Parameterized SF
  - easy to implement
  - control information revelation

# Competitive market: Optimal demand response

- Customer $i$ cost (or disutility) function: $C_i(q_i)$
  - continuous, increasing, and strictly convex
- Competitive market and price-taking customers
- Optimal demand response

$$\max_{b_i} \; pq_i(b_i, p) - C_i(q_i(b_i, p))$$



# Competitive equilibrium

**Theorem:** There exist a unique CE. Moreover, the CE is efficient, i.e., maximizes social welfare:

$$\max_{q} \quad -\sum_{i} C_i(q_i) \quad \text{s.t.} \quad \sum_{i} q_i = d$$

**Corollary (Individual Rantionality):**

Any customer who sheds a positive load receives positive net revenue at the competitive equilibrium, i.e. $\bar{p}\bar{q}_i - C_i(\bar{q}_i) > 0$ for all $i \in \bar{N}$.

# Proof

Proof Idea: Compare the equilibrium condition with the optimality condition (KKT) of the optimization problem.

### Equilibrium

$$\max_{b_i} \; p q_i(b_i, p) - C_i(q_i(b_i, p))$$

$$\sum_i q_i(b_i, q) = d$$

### Social Welfare optimization

$$\max_{q_i} \; -C_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d$$

$$\left(C_i'(q_i(b_i, p)) - p\right)\left(\hat{b}_i - b_i\right) \geq 0, \forall \hat{b}_i \geq 0$$

$$\sum_i q_i(b_i, p) = d$$

$$\left(C_i'(q_i) - p\right)(\hat{q}_i - q_i) \geq 0, \forall \hat{q}_i \geq 0$$

$$\sum_i q_i = d$$

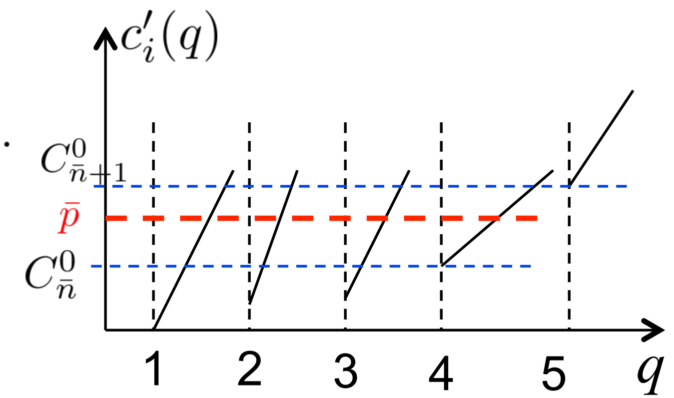
# Competitive equilibrium

Index the customers s.t. $c'_1(0) \le c'_2(0) \le \ldots \le c'_n(0)$. Let $C_i^0 := c'_i(0)$

## Theorem (A water-filling structure):

Let $\{(\bar{b}_i)_{i\in N}, \bar{p}\}$ be a competitive equilibrium and $\bar{q}_i = q_i(\bar{b}_i, \bar{p})$ be the corresponding load shed by $i \in N$. The set of customers that shed a positive load at the equilibrium, i.e. $\{i : \bar{q}_i > 0\}$, is $\bar{N} = \{1, 2, \cdots, \bar{n}\}$ with a unique $\bar{n}$ that satisfies:

$$\sum_i^{\bar{n}} (C'_i)^{-1}(C^0_{\bar{n}}) < d \le \sum_i^{\bar{n}} (C'_i)^{-1}(C^0_{\bar{n}+1}).$$




Moreover, the price $\bar{p}$ satisfies:

$$C^0_{\bar{n}} < \bar{p} \le C^0_{\bar{n}+1}$$

and for any $i \in \bar{N}$, $\bar{p} = C'_i(\bar{q}_i)$.

## Corollary (Individual Rantionality):

Any customer who sheds a positive load receives positive net revenue at the competitive equilibrium, i.e. $\bar{p}\bar{q}_i - C_i(\bar{q}_i) > 0$ for all $i \in \bar{N}$.

# Iterative supply function bidding

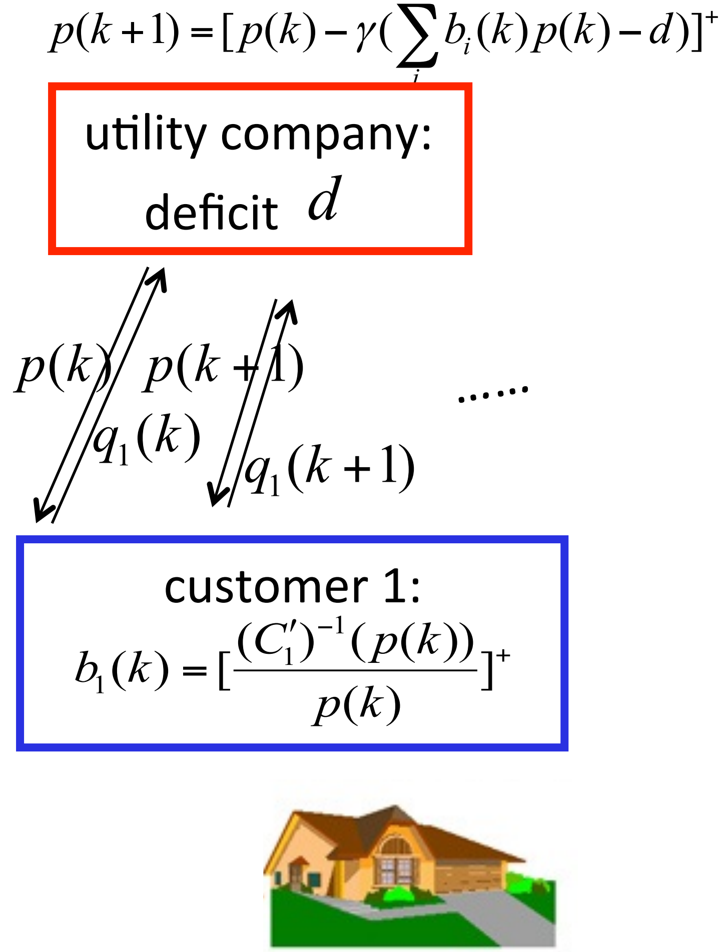
- Upon receiving the price information, each customer $i$ updates its supply function

$$b_i(k) = \left[\frac{(C_i')^{-1}(p(k))}{p(k)}\right]^+$$

- Upon gathering bids from the customers, the utility company updates price

$$p(k+1) = \left[p(k) - \gamma\left(\sum_i b_i(k)p(k) - d\right)\right]^+$$

- Requires
  - timely two-way communication
  - certain computational capability of the customers



# Strategic demand response

- Price-anticipating customer

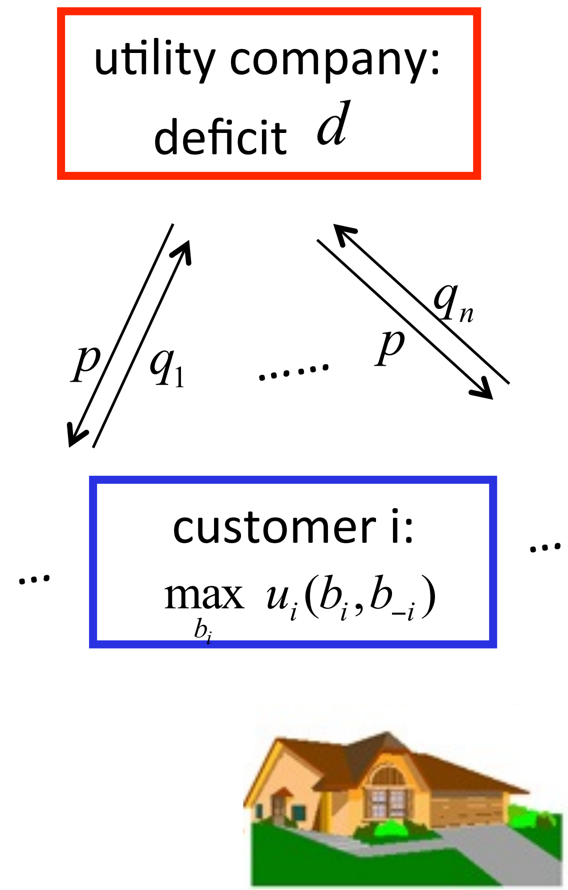
$$\max_{b_i} \; u_i(b_i, b_{-i})$$

with

$$u_i(b_i, b_{-i}) = p(b)q_i(b_i, p(b)) - C_i(q_i(b_i, p(b)))$$

- **Definition**: A supply function profile $b^*$ is a Nash equilibrium (NE) if, for all customers $i$ and $b_i \geq 0$,

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*)$$



# Nash Equilibrium

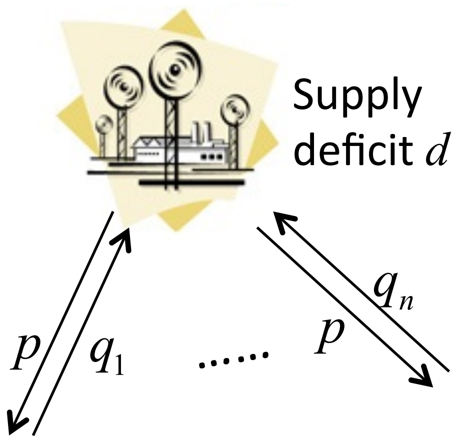
- Price-anticipating customer

$$\max_{b_i} \; p(b_i,b_{-i})q_i(b_i, p(b_i,b_{-i})) - C_i(q_i(b_i, p(b_i,b_{-i})))$$

- Nash equilibrium exits and is unique when the number of customers is larger than 2
- Each customer will shed a load of less than d/2 at the equilibrium
- Solving another global optimization problem

$$\max_{0\le q_i \le d/2} \; -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d$$

$$D_i(q_i) = (1+\frac{q_i}{d-2q_i})C_i(q_i) - \int_0^{q_i} \frac{d}{(d\text{-}2x_i)^2} C_i(x_i)dx_i$$




customer i:

$$\max_{b_i} \; p(b_i,b_{-i})q_i(b_i, p(b_i,b_{-i})) - C_i(q_i(b_i, p(b_i,b_{-i})))$$

# Nash equilibrium

**Theorem**

Assume $|N| \geq 3$. The demand response game has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\min_{0 \leq q_i < d/2} \quad \sum_i D_i(q_i)$$

$$s.t. \quad \sum_i q_i = d,$$

with

$$D_i(q_i) = (1 + \frac{q_i}{d - 2q_i})C_i(q_i) - \int_0^{q_i} \frac{d}{(d - 2x_i)^2} C_i(x_i) dx_i.$$

# Proof

Proof Idea: Compare the equilibrium condition with the optimality condition (KKT) of the optimization problem.

**NE Equilibrium**

$$\max_{b_i} \quad p q_i(p(b), p) - C_i(q_i(p(b), p))$$

$$= d^2 b_i / (\Sigma_j b_j)^2 - C_i(d b_i / \Sigma_j b_j)$$

$$\sum_i q_i(b_i, q) = d$$

**Optimization**

$$\max_{q_i} \quad -D_i(q_i) \quad \text{s.t.} \quad \sum_i q_i = d$$

$$D_i(q_i) = (1 + q_i / d - 2q_i) C_i(q_i)$$

$$- \int_0^{q_i} d / (d - 2x_i)^2 C_i(x_i) dx_i$$

$$\left(\frac{d}{B^*_{-i} + b^*_i} - \frac{B^*_{-i}}{B^*_{-i} - b^*_i} C'_i\left(\frac{d b^*_i}{B^*_{-i} + b^*_i}\right)\right)(b_i - b^*_i) \leq 0$$

$$p^* = \frac{d}{\sum_i b^*_i}$$

$$q^*_i = b^*_i p^*$$

$$B_i = \sum_{j \neq i} b_j$$

$$\left(p^* - \left(1 + \frac{q^*_i}{d - 2q^*_i}\right) C'_i(q^*_i)\right)(q_i - q^*_i) \leq 0$$

$$\forall q_i \geq 0$$

$$\sum_i b^*_i p^* = d$$

$$p^* \geq 0$$

# Nash equilibrium

## Theorem (A water-filling structure):

Assume $|N| \geq 3$. Let $\{(b_i^*)_{i \in N}\}$ be a Nash equilibrium, $p^* = d/\sum_i b_i^*$ be the Nash equilibrium price, and $q_i^* = (b_i^*, p^*)$ be the corresponding load shed by $i \in N$. The set of customers that shed a positive load at the Nash equilibrium, i.e. $\{i : q_i^* > 0\}$, is $N^* = \{1, 2, \cdots, n^*\}$ with a unique $n^*$ that satisfies

$$\sum_i^{n^*} (D_i')^{-1}(C_{n^*}^0) < d \leq \sum_i^{n^*} (D_i')^{-1}(C_{n^*+1}^0) \tag{1}$$

Moreover, the price $p^*$ satisfies

$$C_{n^*}^0 < p^* \leq C_{n^*+1}^0. \tag{2}$$

and for any $i \in N^*$, $p^* = D_i'(q_i^*)$.

## Corollary (Individual Rationality):

Any customer who shed a positive load at the Nash equilibrium receives positive net revenue, i.e. $p^* q_i^* - C_i(q_i^*) > 0$ for all $i \in N^*$.

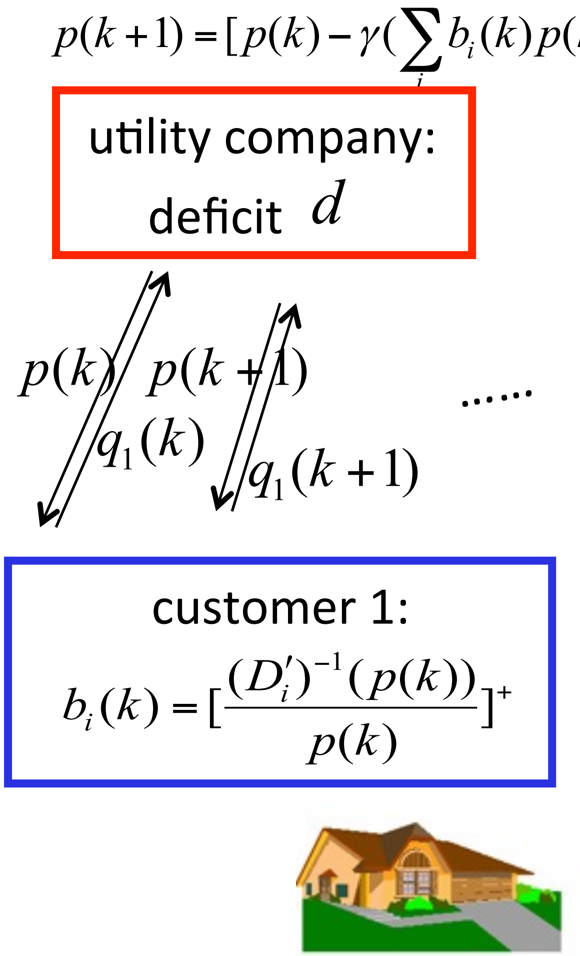
# Iterative supply function bidding

- Each customer $i$ updates its supply function

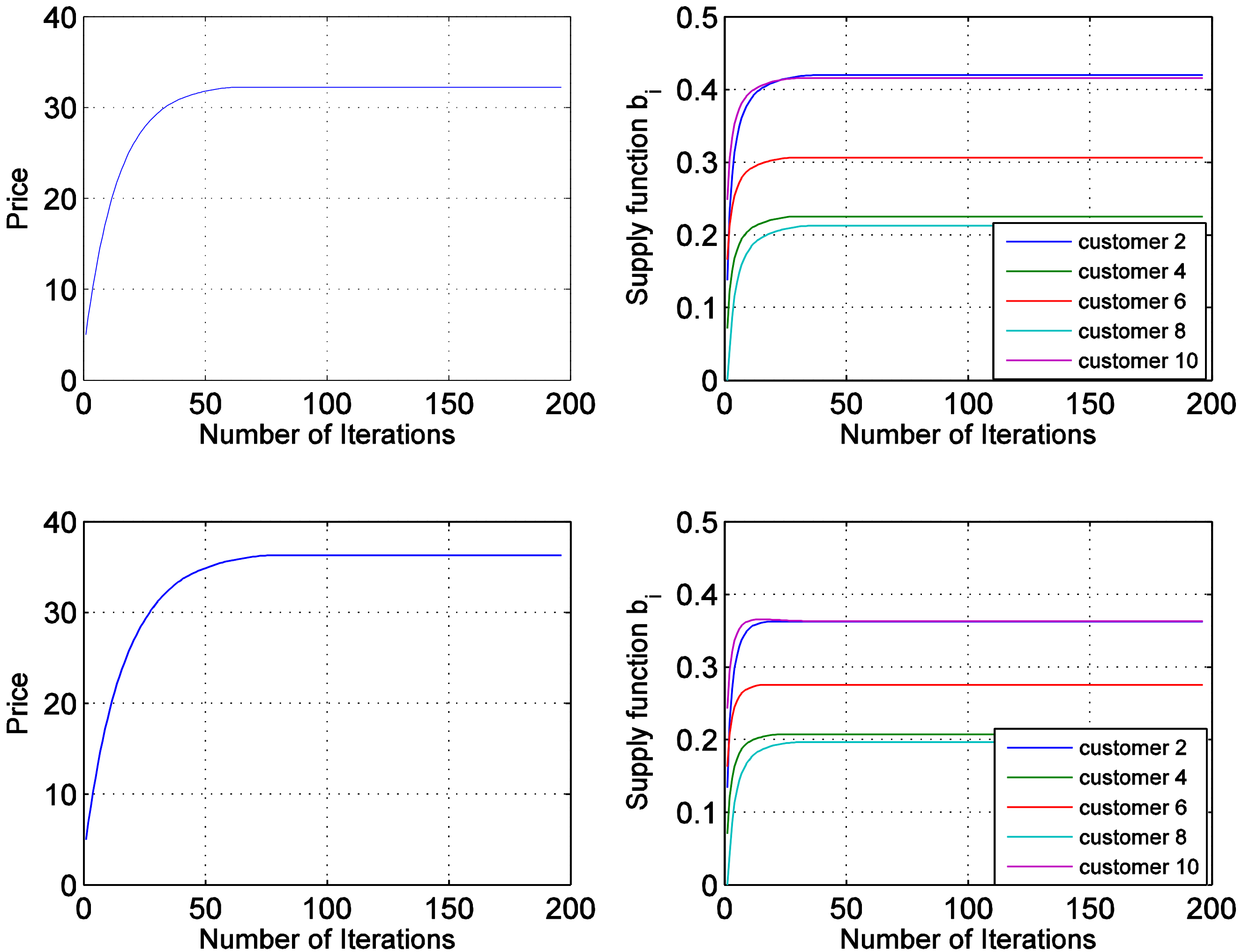
$$b_i(k) = [\frac{(D_i')^{-1}(p(k))}{p(k)}]^+$$

- The utility company updates price

$$p(k+1) = [p(k) - \gamma(\sum_i b_i(k)p(k) - d)]^+$$



# Numerical example




Optimal supply function bidding (upper panels) v.s. strategic bidding (lower panels)

# Efficiency Loss of NE (Price of Anarchy)

**Theorem:**

Let $\{(\bar{b}_i)_{i \in N}, \bar{p}\}$ be a competitive equilibrium, $\{(b_i^*)_{i \in N}\}$ be a Nash equilibrium and $p^*$ be the corresponding price at the Nash equilibrium, we have:

1. $\bar{N} \subseteq N^*$, where $\bar{N} := \{i : \bar{q}_i := b_i \bar{p} > 0\}$ is the set of customers who shed a positive load at the competitive equilibrium; and $N^* := \{q_i^* := b_i^* p^* > 0\}$ is the set of customers who shed a positive load at the Nash equilibrium.

2. $\bar{p} \leq p^* \leq \frac{n-1}{n-2} \frac{M}{m} \bar{p}$, where $M := \max_{i \in N} C_i'(\frac{d}{n})$; $m := \min_{i \in N} C_i'(\frac{d}{n})$.

3. $\bar{C} \leq C^*$, and if we further assume $\bar{q}_{\max} := \max_i \bar{q}_i < \frac{d}{2}$, then we have

$$C^* \leq (1 + \frac{\bar{q}_{\max}}{d - 2\bar{q}_{\max}})\bar{C}.$$

   Here $\bar{C} = \sum_i C_i(\bar{q}_i)$ be the total social cost at the competitive equilibrium and $C^* = \sum_i C_i(q_i^*)$ is the total cost at the Nash equilibrium.

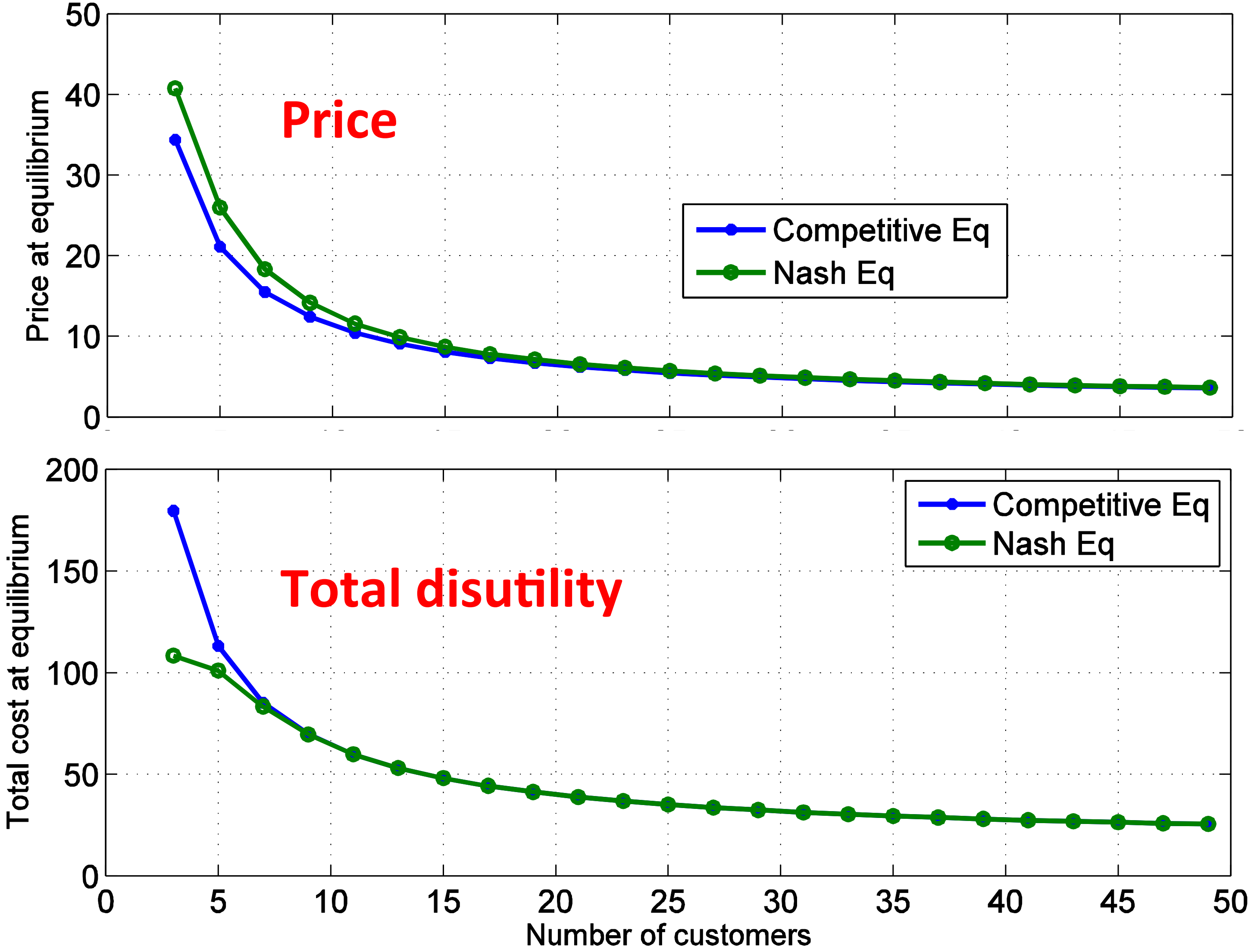
# Homogeneous Customers

**Corollary:**

Assume that all the customers have a same cost function. Then we have

1. $\bar{p} \leq p^* \leq \frac{n-1}{n-2}\bar{p}$. As $n \to \infty$, $p^* \to \bar{p}$
2. $\bar{C} \leq C^* \leq \frac{n-1}{n-2}\bar{C}$. As $n \to \infty$,$C^* \to \bar{C}$.

# Numerical example



# A Special Case with Quadratic Disutility Function

**Theorem:**

Suppose each customer has a quadratic cost function, i.e. $C_i(q) = \frac{1}{2} c_i q^2$ for each $i$.

1. $\{\left(\bar{b_i}\right)_{i \in N}, \bar{p}\}$ is a competitive equilibrium if and only if $\bar{b}_i = \frac{1}{c_i}$.
2. $\{(b_i^*)_{i \in N}\}$ is a Nash equilibrium if and only if $\{(b_i^*)_{i \in N}\}$ satisfies the following equalities,

$$b_i^* = (1 - c_i b_i^*) B_{-i}^*, \forall i \in N.$$

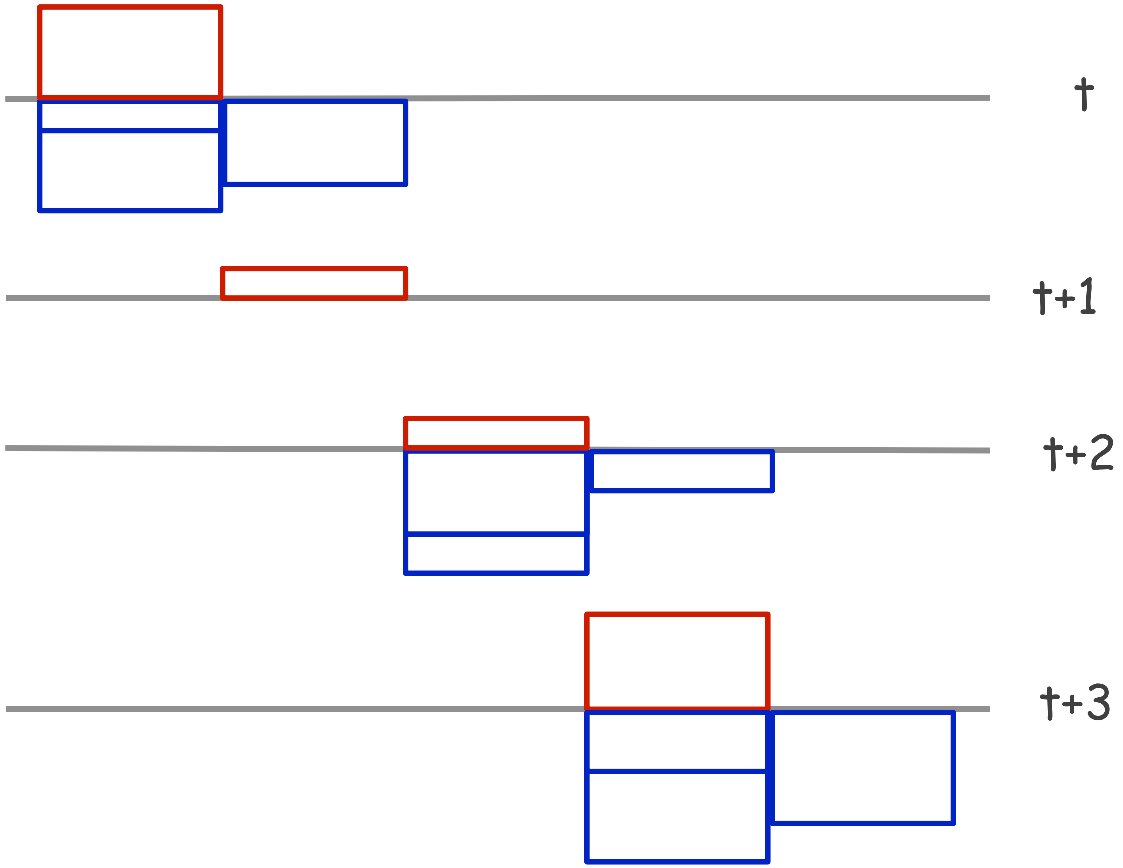
**Message:**

Both the competitive equilibrium and game equilibrium **are independent of** the supply deficit *d*!

# Value of Anarchy

- Price of Anarchy: Loss in efficiency due to strategic interactions in contrast to a coordination
- Simple model: one agent with shiftable demand and another with instantaneous demand
- Contrast optimal efficient solution to a Stackelberg game of strategic behavior
- A new tradeoff: Cooperation can increase endogenous risk

# Setup




Inflexible load

Flexible load

# Model

**System state:**

Aggregate unshiftable loads $x(t)$

$$\underbrace{x(t)}_{\text{aggregate unshiftable}} = \underbrace{d_1(t)}_{\text{unshiftable arrival at current period}} + \underbrace{d_2(t-1) - u(t-1)}_{\text{leftover from last period's shiftable}}$$

Consumer arrival with shiftable load $d_2(t)$

**Load shifting decision:**

Only 1 decision maker at $t$ : the new arrival with shiftable load
Split load into two periods $(t, t+1)$ based on $(x(t), d_2(t))$

$$(u(t), d_2(t) - u(t))$$

# Problem Formulation

- Deadline constraints on demands:

$$\sum_{t \text{ in } i\text{'th active window}} u_{t,i} = i\text{'th work load}$$

- Endogenous prices couple individual decisions:

$$p_t \propto \sum_i u_{t,i}$$

- **Non-cooperative** decision making:

$$\min_{u_{t,i}} p_t u_{t,i} + \mathbb{E}[p_{t+1} u_{t+1,i}]$$

Minimize individual cost

- **Cooperative** decision making:

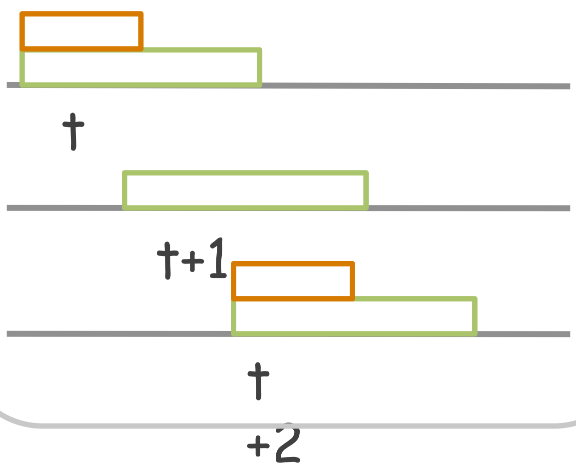
$$\min_{[u_{t,i}]_{t,i}} \mathbb{E}\left[\text{time average of } \sum_i p_t u_{t,i}\right]$$

Minimize aggregate cost

# Solution: Strategic

**Symmetric Markov Perfect equilibrium in dynamic stochastic game**

$$u^s(x(t), d_2(t)) = \arg\min_u \{p(t)u + \mathbf{E}_t[p(t+1)(d_2(t) - u)]\}$$




$$p(t) = x(t) + u$$
$$p(t+1) = x(t+1) + u^s(x(t+1), d_2(t+1))$$

Overlapping type 2 consumers

Flavor of Stackelberg competition

# Solution: Strategic

**Symmetric Markov Perfect equilibrium in dynamic stochastic game**

$$u^s(x(t), d_2(t)) = \arg\min_u \{p(t)u + \mathbf{E}_t[p(t+1)(d_2(t) - u)]\}$$

**Equilibrium strategy**

Unique MPE with linear stationary equilibrium strategy:

$$u^s(x, d_2) = -\underbrace{\frac{1}{2(1+\sqrt{1-\frac{q_2}{2}})}}_{a^s} x + \underbrace{\frac{1}{1+\frac{1}{\sqrt{1-\frac{q_2}{2}}}}}_{b^s} d_2 + \underbrace{\frac{q_1\mu_1 + q_2\mu_2 \frac{1}{1+\sqrt{1-\frac{q_2}{2}}}}{2(1+\sqrt{1-\frac{q_2}{2}})}}_{e^s}$$

# Solution: Cooperative

**Bellman equation for infinite horizon average cost MDP**

$$\lambda^c + V^c(x) = (1 - q_2)(x^2 + \mathbf{E}[V^c(d_1)]) + q_2\mathbf{E}[\min_u\{(x+u)^2 + V^c(d_2 - u + d_1)\}]$$

**Optimal stationary policy**

There exists an optimal linear stationary policy:

$$u^c(x, d_2) = -\underbrace{\frac{1}{1+\sqrt{1-q_2}}}_{a^c}x + \underbrace{\frac{1}{1+\frac{1}{\sqrt{1-q_2}}}}_{b^c}d_2 + \underbrace{\frac{q_1\mu_1 + q_2\mu_2\frac{1}{1+\sqrt{1-q_2}}}{1+\sqrt{1-q_2}}}_{e^c}$$

# Welfare impacts

Under linear stationary policy $u(x, d_2) = -ax + bd_2 + e$

## Efficiency/Welfare

**Variance** $-\frac{1}{2}\mathbf{E}[U(t)^2] = -\frac{1}{2}\lambda$

## Risk

**Tail probability** $\Pr(x(t) \geq M)$

$$\mathcal{X} = \mathcal{X}_k \text{ with probability } \mathrm{q}_2^{\mathrm{k}}(1 - \mathrm{q}_2)$$

$$\mathbf{E}[\mathcal{X}_k] = \frac{(1 - a^{k+1}\mu_1 + (1 - a^k)((1 - b)\mu_2 - e))}{1 - a}$$

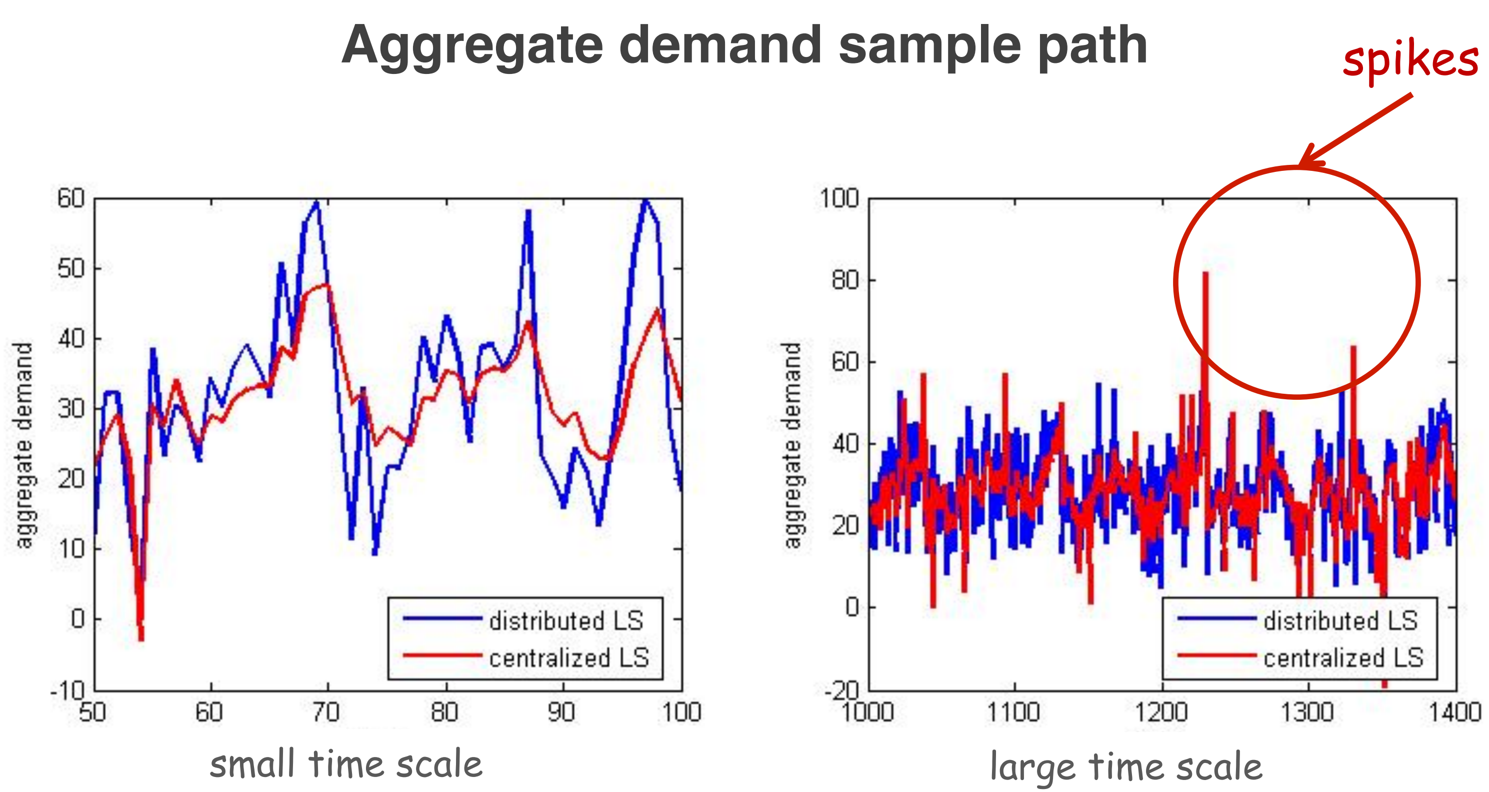
$$Var[\mathcal{X}_k] = \frac{(1 - a^{2(k+1)})\sigma_1^2 + (1 - a^{2k})(1 - b)^2\sigma_2^2}{1 - a^2}$$

**Strategic** $u^s(x, d_2) = -a^s x + b^s d_2 + e^s$

$\wedge \qquad \vee$

**Cooperative** $u^c(x, d_2) = -a^c x + b^c d_2 + e^c$

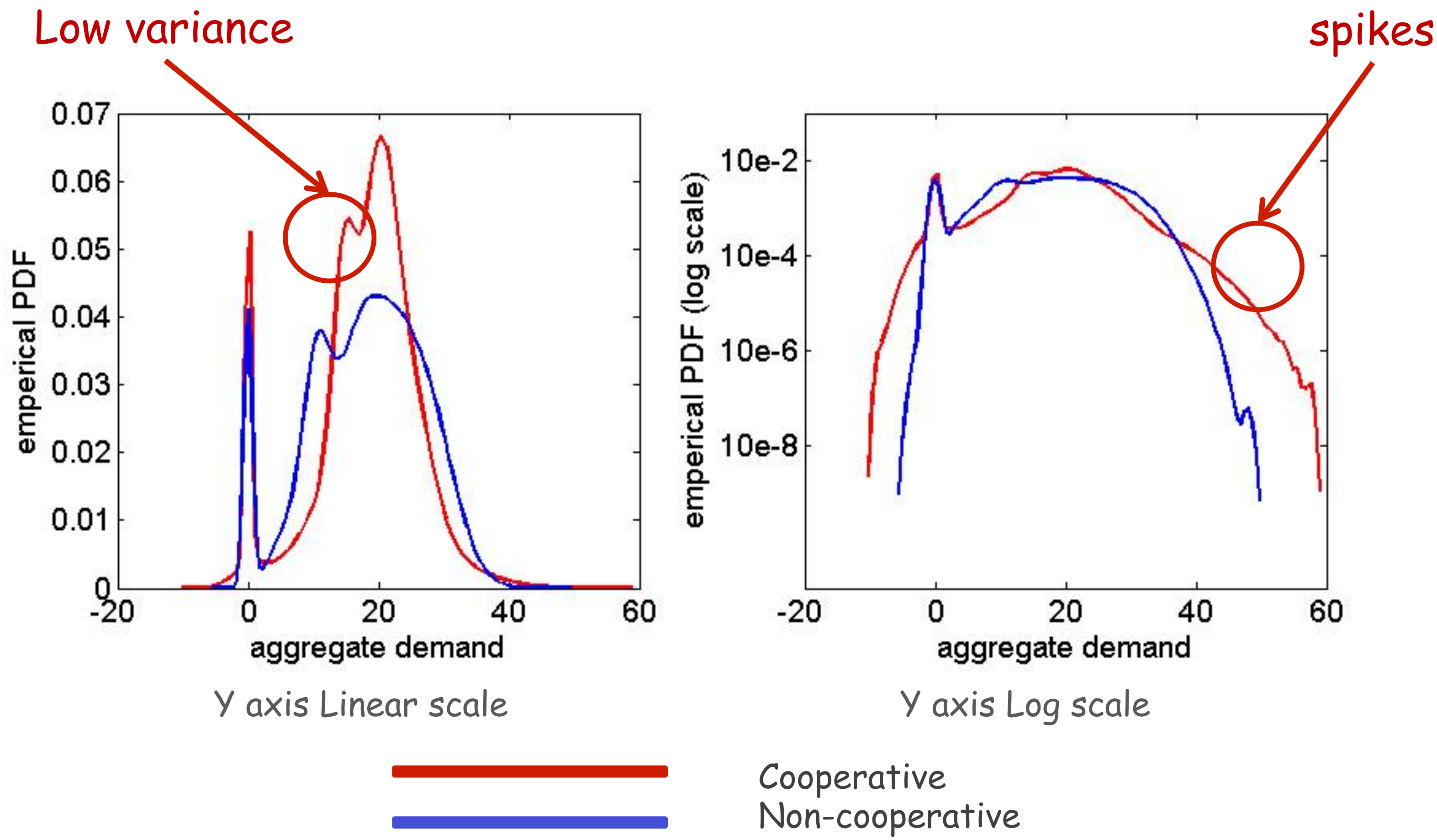
# Price of Anarchy: what about risk?




Cooperative
Non-cooperative

# Example I: L = 2

**Aggregate demand stationary distribution**



# Concluding remarks

- Studied one abstract models for demand response
  - Characterized competitive as well as strategic equilibria
  - Proposed distributed demand response algorithms based on optimization problem characterizations
  - Characterized the efficiency loss and price of the game-theoretic equilibrium
- Risk Analysis:
  - Performance-robustness Tradeoffs
  - Market Mechanism

**Thank you!**